

# YEAR 12 *Trial Exam Paper*

## Mathematical Methods

### Written Examination 1

### Question and Answer Book

### 2024 Insight Year 12 Trial Exam Paper

- **Reading time:** 15 minutes
- **Writing time:** 1 hour
  
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

#### Materials supplied

- Question and Answer Book of 15 pages
- Formula Sheet
- Working space is provided throughout the book.

#### Instructions

- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

#### At the end of the examination

- You may keep the Formula Sheet.

Students are **not** permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

---

**Number of questions:** 9

**Number of questions to be answered:** 9

**Number of marks:** 40

This trial examination produced by Insight Publications is NOT an official VCAA paper for the 2024 Year 12 Mathematical Methods Written Examination 1. The Publishers assume no legal liability for the opinions, ideas or statements contained in this trial examination. This examination paper is licensed to be printed, photocopied or placed on the school intranet and used only within the confines of the purchasing school for examining their students. No trial examination or part thereof may be issued or passed on to any other party, including other schools, practising or non-practising teachers, tutors, parents, websites or publishing agencies, without the written consent of Insight Publications.

Insight Publications has made every effort to meet VCAA's updated accessible font and layout specifications as of the information available on 08/02/2024.

Copyright © Insight Publications 2024

**This page is blank.**

**Instructions**

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

---

**Question 1** (3 marks)

a. Let  $y = x \cos(2x)$ .

Find  $\frac{dy}{dx}$ .

1 mark

---

---

---

b. Let  $f(x) = \frac{\log_e(x)}{e^x - 1}$ .

Find and simplify  $f'(1)$ .

2 marks

---

---

---

---

**Question 2** (2 marks)Solve  $2\sin^2(x) + 3\sin(x) - 2 = 0$ , where  $x \in [0, 2\pi]$ .

2 marks

---

---

---

---

---

---

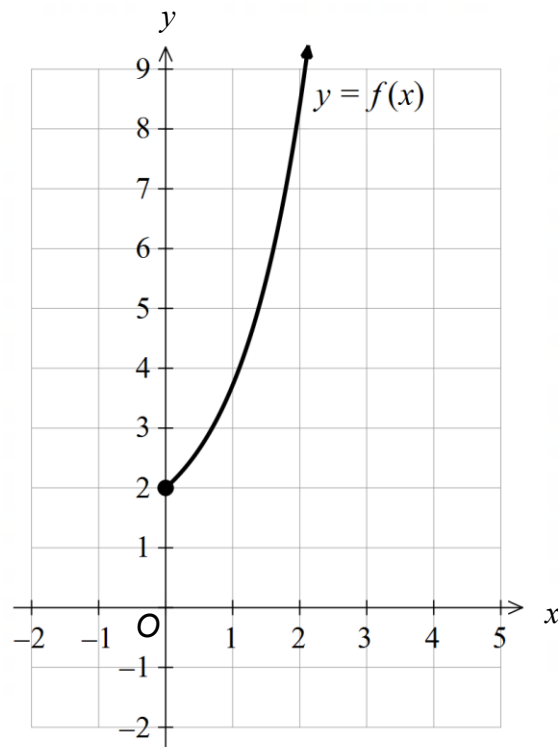
---

**This page is blank.**

**Question 3** (6 marks)

Let  $f : [0, \infty) \rightarrow \mathbb{R}$ ,  $f(x) = e^x + 1$ .

The graph of  $y = f(x)$  is shown below over part of its domain.



- a. Use two trapeziums of equal width to approximate the area between the curve, the  $x$ -axis and the lines  $x=0$  and  $x=2$ .

2 marks

---

---

---

---

Let  $g : [0, \infty) \rightarrow \mathbb{R}$ ,  $g(x) = 4e^{-x} + 1$ .

- b.** Find the  $x$ -coordinate of the point where the graphs of  $y = f(x)$  and  $y = g(x)$  intersect. Express your answer in the form  $x = \log_e(a)$ , where  $a \in \mathbb{R}^+$ .

2 marks

---



---



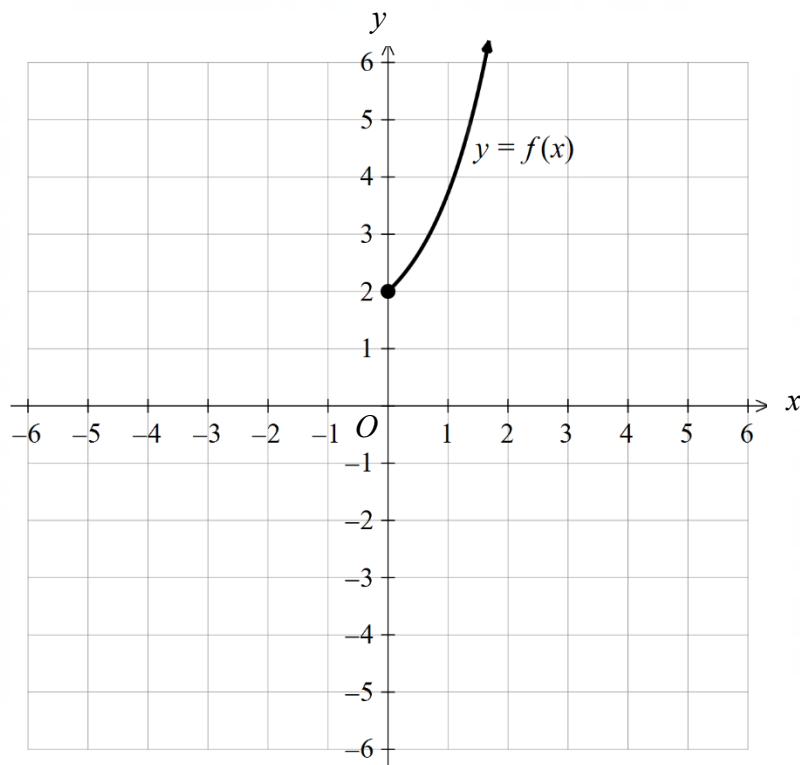
---



---

- c.** Part of the graph of  $y = f(x)$  is shown below. Sketch the graph of  $y = g(x)$  on the same axes. Label any asymptotes with their equation, and any end points and/or axis intercepts with their coordinates.

2 marks



**Question 4** (5 marks)

Let  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = 2\cos(4x) + 1$ .

a. State the range of  $f$ .

1 mark

---

Let  $g: [0, a] \rightarrow \mathbb{R}$ ,  $g(x) = 2\cos(4x) + 1$  and  $h: [0, \infty) \rightarrow \mathbb{R}$ ,  $h(x) = \sqrt{x}$ .

b. i. Find the largest possible value of  $a$ , such that  $(h \circ g)(x)$  exists.

2 marks

---

---

---

---

---

---

ii. If  $a = \frac{\pi}{8}$ , state the range of  $(h \circ g)(x)$ .

2 marks

---

---

---

---

---



**Question 5** (4 marks)

Let  $f : \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = x^3 + x^2$ .

- a. Determine the  $x$ -coordinates of the stationary points of  $f$  and state the nature of each stationary point.

2 marks

---

---

---

---

---

- b. Find the coordinates of the point of inflection of  $f$ .

2 marks

---

---

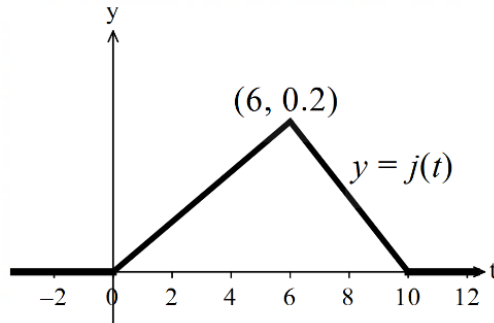
---

---

**Question 6** (4 marks)

The length of time,  $t$  hours, that Jia uses her laptop on any given day is a continuous random variable, with probability density function  $j(t)$ .

The graph of  $y = j(t)$  is shown below.



The maximum value of  $j(t)$  occurs when  $x = 6$ . Further,  $j(t) = 0$  when  $t \leq 0$  and  $t \geq 10$ .

- a. Find the probability that Jia uses her laptop for more than 6 hours on a given day.

1 mark

---



---

- b. The length of time,  $X$  hours, that Khan uses his laptop on any given day is a continuous random variable. The probability density function of  $X$  is given by

$$k(x) = \begin{cases} \frac{x+1}{12} & 0 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

Find the value of  $w$ , such that  $\Pr(X \leq w) = \frac{1}{3}$ .

3 marks

---



---



---



---



---



---

**Question 7** (6 marks)

Let  $g : \left[\frac{3}{2}, \infty\right) \rightarrow \mathbb{R}$ ,  $g(x) = \sqrt{2x-3}$ .

- a. Show that  $g'(x) = \frac{1}{\sqrt{2x-3}}$ .

1 mark

---

---

---

- b. Find the angle from the positive direction of the  $x$ -axis to the tangent of the graph of  $g$  at  $x = 2$ , measured in an anticlockwise direction. State your answer in degrees.

2 marks

---

---

---

- c. Consider the angle from the positive direction of the  $x$ -axis to the tangent of the graph of  $g$  at  $x = k$ , measured in an anticlockwise direction. Find the set of values of  $k$  for which this angle is at least  $30^\circ$ .

3 marks

---

---

---

---

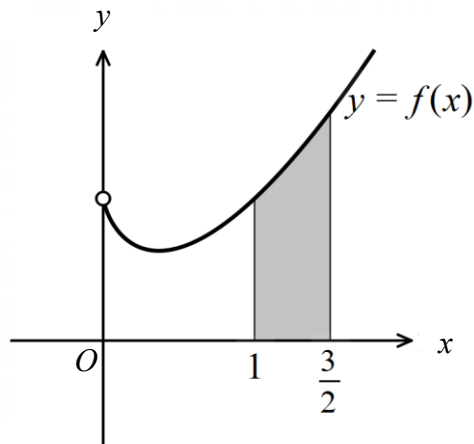
---

---

**Question 8** (4 marks)

Let  $f : \mathbb{R}^+ \rightarrow \mathbb{R}$ ,  $f(x) = x \log_e(x) + 1$ .

Part of the graph of  $y = f(x)$  is shown below.



- a. Show that  $\frac{d}{dx}(x^2 \log_e(x)) = 2x \log_e(x) + x$ .

1 mark

---

---

---





- c. The restaurant also sells jars of chilli sauce. The volume of jars of chilli sauce can be represented by the variable  $V$ , which is normally distributed with a mean of 205 g and a standard deviation of 3 g.

If  $Z$  is the standard normal random variable,  $\Pr(Z < -2) = a$  and  $\Pr(-2 < Z < -1) = b$ , express  $\Pr(V > 202 | V < 211)$  in terms of  $a$  and  $b$ .

2 marks

---

---

---

---

---

---

---