The Mathematical Association of Victoria

Trial Exam 2024

MATHEMATICAL METHODS

WRITTEN EXAMINATION 1

STUDENT NAME ____

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software) or notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 11 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (3 marks)

a. Let $y = \frac{\log_e(x)}{x^2}$, where $x > 0$.

Find and simplify $\frac{dy}{dx}$. 1 mark

b. If $g(x) = x\tan^2(x)$, find $g'\left(\frac{\pi}{4}\right)$. 2 marks

TURN OVER

Question 2 (3 marks)

Solve $\log_e\left(3x^2-1\right)-\log_e\left(1-3x\right)=\log_e(2)$ for x.

Question 3 (3 marks)

Consider the following simultaneous equations

$$-2x+ky=m$$

$$\left(1+k^2\right)x+y=2$$ where k and m are real constants.

Determine the values of k and m for which there are no solutions.

Question 4 (3 marks)

Solve $2\sin^2(x)+\sin(x)=1$ for $x\in[-\pi,\pi]$.

Question 5 (3 marks)

Let $f:R\setminus\{2\}\to R,\ f(x)=\dfrac{1}{(2-x)^2}$ and $g:[-2,k)\to R,\ g(x)=2x+1$, where k is a real constant.

a. If $k=1$, explain why $f\circ g$ does not exist? 1 mark

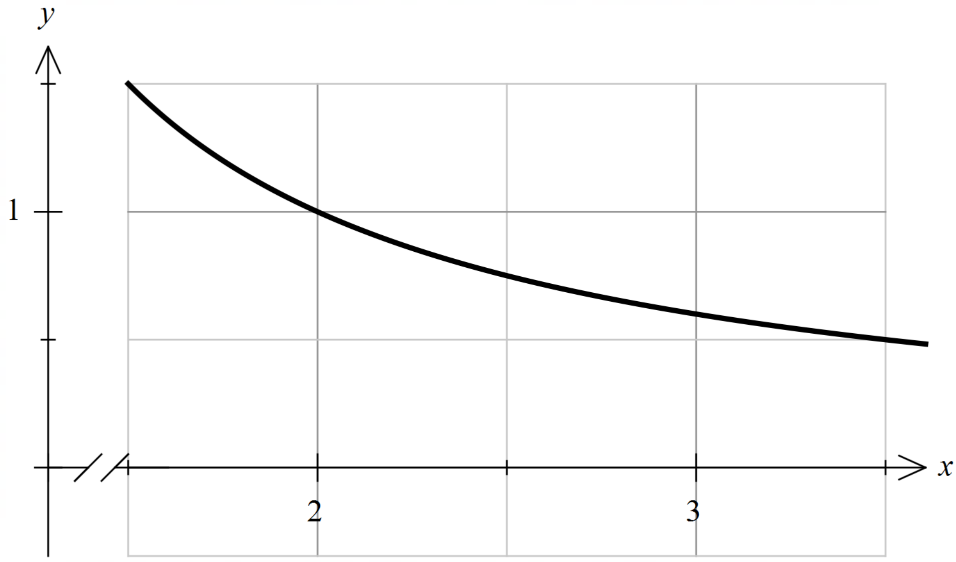
b. State the maximum possible value of k such that $f\circ g$ exists. 1 mark

c. Define $f\circ g$ for the value of k in **part b**. 1 mark

TURN OVER

Question 6 (5 marks)

Part of the graph of $y=\frac{3}{2x-1}$ is shown below.



a. Evaluate $\int_{2}^{3}\frac{3}{2x-1}dx$, giving your answer in the form $a\log_e(b)$ where $a,b\in Q$. 2 marks

b. Using two trapeziums of equal width, find the approximate area between the curve, the x-axis and the lines $x=2$ and $x=3$. 2 marks

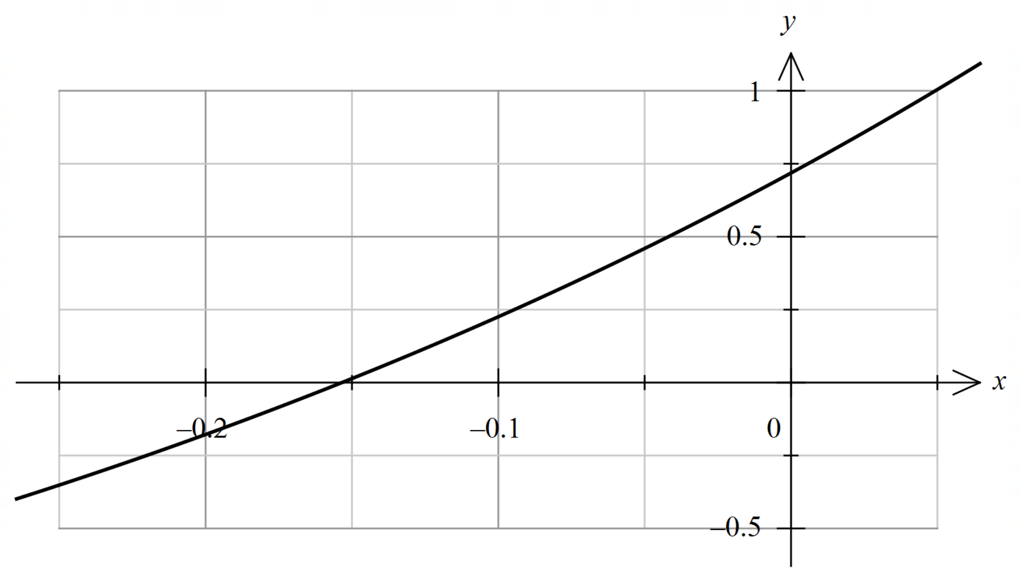
c. Will the area in **part b.** be an over or under estimate of the actual area? Explain. 1 mark

Question 7 (4 marks)
Consider the x-intercept of the function $f(x) = e^{2x+1} - 2$.

a. Find x_1 using Newton's method with an initial estimate of $x_0 = 0$. 1 mark

Part of the graph of f is shown below.

b. Sketch the tangent line to the curve at $x = 0$ and label x_1. 1 mark



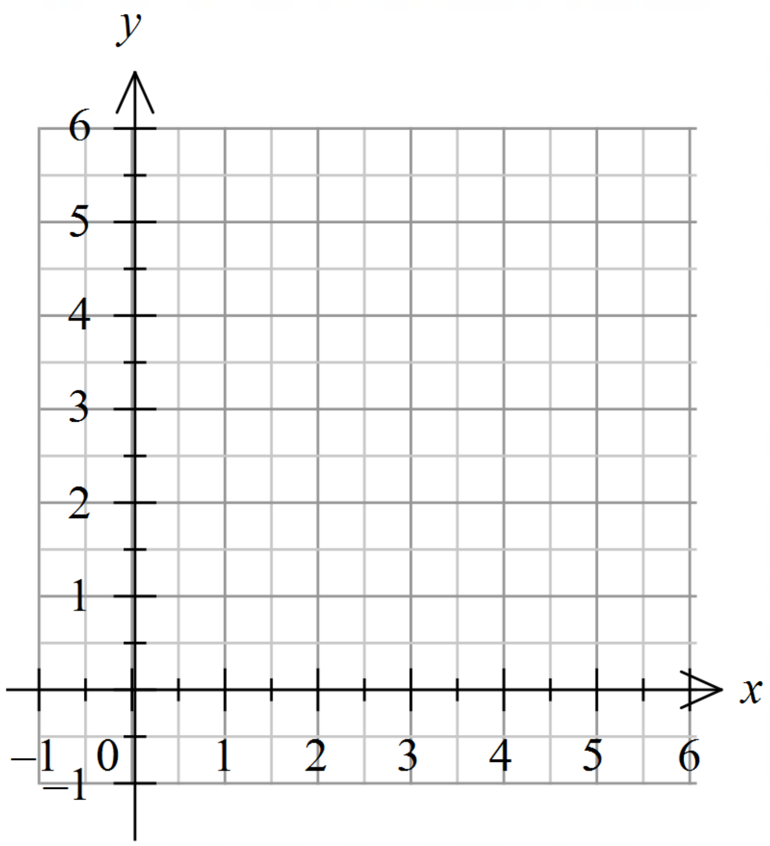
c. What is the distance between x_1 and the exact value of the x-intercept?
Express your answer in the form $a + \log_e(b)$ where $a, b \in R$. 2 marks

Question 8 (8 marks)

Consider the function f with rule $f(x)=1+\sqrt{2x-3}$ over its maximal domain.

a. Give the coordinates of the point where $f'(x)=1$. 1 mark

b. Sketch the graphs of $y=f(x)$ and $y=f^{-1}(x)$ on the set of axes below.
Label the endpoints and point of intersection with their coordinates. 3 marks



Question 8 - continued

Now consider the family of curves, over their maximal domains, with rule $g(x)=1+\sqrt{2x-a}$, where a is a real constant.

c. Find the values of a for which the graphs of g and g^{-1} have two points of intersection. 1 mark

d. Find the area bound by the graphs of g and g^{-1} when $a=2$. 3 marks

TURN OVER

Question 9 (8 marks)
The probability Lisa hits a bullseye in a game of darts at the end of a workday is 0.02. The probability she hits a bullseye on a non workday is 0.3. Lisa is a school teacher and works everyday from Monday to Friday but not on the weekends. Each throw is independent of each other.

a. What is the probability Lisa hits three bullseyes in a row on a Wednesday night? 1 mark

b. What is the probability she hits exactly three bullseyes out of four throws on a Saturday? 2 marks

c. Lisa has five throws on a Friday night. Given that she throws three bullseyes on her first three throws, what is the probability she throws exactly four bullseyes? 1 mark

Let n_t represent the number of throws on a particular Thursday evening and n_s represent the number of throws on a Saturday evening.

d. If the mean number of bullseyes Lisa scored on the Thursday evening was the same as the mean number she scored on a Saturday evening, find n_t in terms of n_s, giving a general solution. Assume she had at least one throw on each evening. 1 mark

Question 9 - continued

The time Lisa spends practising her dart throwing each day is a random variable, T hours, with probability density function, d given by

$$d(t)=\begin{cases} t-1 & 1\le t\le 2 \\ a(t-3)^2+b & 2<t\le 3 \\ 0 & \text{elsewhere} \end{cases} \quad \text{where } a \text{ and } b \text{ are real constants.}$$

e. Find a and b if the graph of d is continuous for $-\infty < t \le 3$. 3 marks

END OF QUESTION AND ANSWER BOOK