

2024 VCE Mathematical Methods Year 12 Trial Examination 2



Kilbaha Education

Quality educational content

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Victorian Certificate of Education 2024

STUDENT NUMBER

Figures
Words

Letter

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MATHEMATICAL METHODS Trial Written Examination 2

Reading time: 15 minutes
Total writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
A	20	20	20
B	5	5	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, their full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer booklet of 29 pages.
- Detachable sheet of miscellaneous formulas at the end of this booklet.
- Answer sheet for multiple-choice questions.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Write your **name** and **student number** on your answer sheet for multiple-choice questions, and sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – Multiple-choice questions**Instructions for Section A**

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Mark will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Consider the functions $f : R \rightarrow R$, $f(x) = a + b \sin\left(\frac{\pi(x-a)}{c}\right)$ and

$g : R \rightarrow R$, $g(x) = a - b \cos\left(\frac{\pi x}{c}\right)$, where $a, b, c \in R$, $c \neq 0$. The functions f and g have

- A. the same period, the same amplitudes, and the same range.
- B. the same period, the same amplitudes and different ranges.
- C. different periods, different amplitudes, but have the same range.
- D. different periods, different amplitudes, and different ranges.

Question 2

Given the functions $f(x) = \log_e(b-x)$ and $g(x) = \sqrt{x+b}$ are defined on their maximal domains and $b \in R^+$, then the domain of $\frac{f}{g}$ is

- A. $[-b, b)$
- B. $[-b, b]$
- C. $(-b, b)$
- D. $R \setminus \{b\}$

Question 3

The function f is continuous over $0 \leq x \leq 4a$ then $\int_0^{4a} f(x) dx$ is equal to

- A. $\int_0^{2a} (f(x) + f(4a - x)) dx$
- B. $\int_0^{2a} f(x) dx + \int_{2a}^{4a} f(x - 2a) dx$
- C. $\int_0^{2a} (f(x) - f(x - 4a)) dx$
- D. $2 \int_0^{2a} f(x - 2a) dx$

Question 4

The approximate area bounded the curve $f(x) = \frac{1}{x}$, the x -axis, and the lines $x=1$ and $x=4$ using the trapezoidal rule with three equally spaced strips is

- A. $\frac{35}{12}$
- B. $\frac{35}{24}$
- C. $\frac{25}{24}$
- D. $\log_e(4)$

Question 5

Given that the function $f(x)$ has an inverse function $f^{-1}(x)$. Both functions are continuous and differentiable and over their maximal domains. Then considering the graphs of $y = f(x)$ and $y = f^{-1}(x)$,

Allan stated that if the graphs intersect, then they have only one point of intersection which lies on the line $y = x$.

Ben stated that the graphs could have more than one point of intersection which do not on the line $y = x$.

Colin stated that it is possible for the graphs not to intersect.

Then

- A. All of Allan, Ben and Colin are correct.
- B. Only Ben is correct.
- C. Only Ben and Colin are correct.
- D. Only Allan and Colin are correct.

Question 6

Box A contains $r-1$ red marbles and $b+1$ blue marbles, and box B contains $r+1$ red marbles and $b-1$ blue marbles, where $r, b \in \mathbb{Z}^+$ and $r > 3$ and $b > 3$. Jack chooses a box at random and draws two marbles from the same the box without replacement, the probability that the marbles are different colours is equal to

- A. $\frac{(r-1)(b+1)+(r+1)(b-1)}{2(r+b)(r+b-1)}$
- B. $\frac{2(br-1)}{(r+b)(r+b-1)}$
- C. $\frac{(r-1)(b+1)+(r+1)(b-1)}{(r+b)^2}$
- D. $\frac{r(r+1)+(r-1)(r-2)(b+1)b+(b-1)(b-2)}{2(r+b)(r+b-1)}$

Question 7

The graph of $y = (x-a)^{\frac{1}{n}}$ is differentiable at $x = a$ when

- A. $n = 2$
- B. $n = -3$
- C. $n = -\frac{1}{2}$
- D. $n = \frac{1}{2}$

Question 8

The graph of $y = 4 \tan\left(2\left(x - \frac{\pi}{3}\right)\right)$

- A. crosses the x -axis at $x = \frac{\pi}{6}(3k+2)$ and has vertical asymptotes at $\frac{\pi}{12}(6k-11)$ where $k \in \mathbb{Z}$.
- B. crosses the x -axis at $x = \frac{\pi}{6}(3k-1)$ and has vertical asymptotes at $\frac{\pi}{12}(6k+5)$ where $k \in \mathbb{Z}$.
- C. crosses the x -axis at $x = \frac{\pi}{12}(6k-5)$ and has vertical asymptotes at $\frac{\pi}{6}(3k-1)$ where $k \in \mathbb{Z}$.
- D. crosses the x -axis at $x = \frac{\pi}{12}(6k-11)$ and has vertical asymptotes at $\frac{\pi}{6}(3k+2)$ where $k \in \mathbb{Z}$.

Question 9

The expression $1 - \left[\binom{10}{1} 0.37 \times 0.63^9 + 0.63^{10} \right]$ represents a binomial probability of

- A. more than two successes in ten trials each with a probability of 0.37
- B. more than two successes in ten trials each with a probability of 0.63
- C. at least two successes in ten trials each with a probability of 0.37
- D. at least two successes in ten trials each with a probability of 0.63

Question 10

A 99% confidence interval for a population proportion is equal (a, b) , then a 95% confidence interval for the population proportion is closest to

- A. $(0.88a + 0.12b, 0.12a + 0.88b)$
- B. $(0.88a - 0.12b, 0.12a + 0.88b)$
- C. $(1.31a, 1.31b)$
- D. $(0.76a, 0.76b)$

Question 11

If $f(x) = g(x)e^{g(x)}$ and $g(2) = 4$ and $g'(2) = 3$ then $f'(2)$ is equal to

- A. $6e^4$
- B. $9e^4$
- C. $12e^4$
- D. $15e^4$

Question 12

Let $f(x) = \int g(x) dx$ and $h(x) = \frac{d}{dx}(g(x))$, then

- A. $h(x) = \frac{d}{dx}(f(x))$
- B. $h(x) = \frac{d}{dx}(f'(x))$
- C. $h(x) = \int f(x) dx$
- D. $h(x) = \int f'(x) dx$

Question 13

For random samples of 18 Australians, \hat{P} represents the proportion of Australians who were born overseas. It is known that less than 35% of Australians were born overseas.

Given that $\Pr\left(\hat{P} = \frac{1}{3}\right) = 0.1873$ then $\Pr\left(\hat{P} < \frac{1}{2}\right)$ is closest

- A. 0.979
- B. 0.940
- C. 0.722
- D. 0.333

Question 14

Given that $f(x)$ is a non-zero odd function.

Which of the following is also an odd function?

- A. $f(x^2)\cos(x)$
- B. $f \circ f(x)$
- C. $f(x^3)\sin(x)$
- D. $f(x^2) - f(x)$

Question 15

The random variable X has the following probability distribution.

x	1	2	3
$\Pr(X = x)$	$\frac{p}{2}$	$\frac{p}{3}$	$1 - \frac{5p}{6}$

Which of the following is **false**?

- A. $E(X) = \frac{1}{3}(9 - 4p)$
- B. $0 < p < \frac{6}{5}$
- C. $E(X^2) = \frac{1}{3}(27 - 17p)$
- D. The maximum variance occurs when $p = \frac{3}{5}$

Question 16

The random variable Z has the standard normal distribution, with $\Pr(0 < Z < a) = A$ and $\Pr(Z > b) = B$ where $0 < a < b < 3$. Then $\Pr(-b < Z < -a \mid Z < 0)$ is equal to

- A. $0.5 - (A + B)$
- B. $0.5 + A - B$
- C. $1 - 2(A + B)$
- D. $1 - \left(\frac{A + B}{2}\right)$

Question 17

Several students were considering the following simultaneous equations

$$3x - 2y + z = 1$$

$$-x + y - z = 2$$

Allan stated that the general solution could be expressed as

$$x = k, \quad y = 2k - 3, \quad z = k - 5 \quad \text{for } k \in \mathbb{R}.$$

Ben stated that the general solution could be expressed as

$$x = \frac{k + 3}{2}, \quad y = k, \quad z = \frac{k - 7}{2} \quad \text{for } k \in \mathbb{R}.$$

Colin stated that the general solution could be expressed as

$$x = k + 5, \quad y = 2k + 7, \quad z = k \quad \text{for } k \in \mathbb{R}.$$

Then

- A. Only Allan is correct.
- B. Only Ben is correct
- C. Only Colin is correct
- D. All of Allan, Ben and Colin are correct.

Question 18

Newton's method is used to solve the equation $f(x) = 0$.

Newton's method can sometimes fail to find the root of the equation $f(x) = 0$.

All of the functions below have a root close to $x = 1$, with a starting value of $x_0 = 2$.

Newton's method will succeed for which one of the following functions?

- A. $f(x) = \sqrt{3x^2 - 7x + 2}$
- B. $f(x) = 3x^3 - 13x^2 + 16x - 4$
- C. $f(x) = (3x - 1)\log_e(x - 2)$
- D. $f(x) = (3x - 1)e^{x-2}$

Question 19

The algorithm below, described in pseudocode, solves the equation $f(x) = 0$ using the bisection method, having a maximum number of iterations.

Inputs: $f(x)$, the function to solve equal to zero
 x_{left} , the initial left estimate of the x -intercept of $f(x)$,
 x_{right} , the initial right estimate of the x -intercept of $f(x)$,

Constants: maxiter , the maximum number of iterations

Define $\text{bisection}(f(x), x_{\text{left}}, x_{\text{right}}, \text{maxiter})$

If $f(x_{\text{left}}).f(x_{\text{right}}) > 0$ **Then**

Print “Starting values incorrect, will not converge”

Print “Require $f(x_{\text{left}}).f(x_{\text{right}}) < 0$ ”

Stop

EndIf

$i \leftarrow 0$

While $i < \text{maxiter}$ **Do**

$x_{\text{mid}} \leftarrow \frac{x_{\text{left}} + x_{\text{right}}}{2}$

If $f(x_{\text{left}}).f(x_{\text{mid}}) < 0$ **Then**

$x_{\text{right}} \leftarrow x_{\text{mid}}$

Else

$x_{\text{left}} \leftarrow x_{\text{mid}}$

EndIf

$i \leftarrow i + 1$

EndWhile

Return x_{mid}

The algorithm is implemented as follows, $\text{bisection}(x^2 - 5, 2, 3, 4)$.

What value would be returned when the algorithm is implemented.

- A. 2.5
- B. 2.125
- C. 2.1875
- D. 2.2188

Question 20

If $\sin^2(x) = \frac{a}{c}$ and $\cos^2(y) = \frac{b}{c}$ where $a, b, c \in \mathbb{R}$, and $0 < a < b < c$ and $x, y \in \left(0, \frac{\pi}{2}\right)$

then $\log_2(\sin(x)\cos(y))$

- A. $\frac{1}{2}(\log_2(a) + \log_2(b)) - \log_2(c)$
- B. $\frac{1}{2}\log_2(ab) + \log_2(c)$
- C. $\log_2(a) + \log_2(b) - \log_2(c)$
- D. $\log_2\left(\sqrt{\frac{a}{c}}\right) + \log_2\left(\sqrt{\frac{b}{c}}\right)$

END OF SECTION A

SECTION B**Instructions for Section B**

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact answer must be given unless otherwise specified. In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (10 marks)

Consider the function $f : R \rightarrow R$, $f(x) = x^4 - 4x^3 + 3$

- a. Find and classify the coordinates of the turning points on the graph of f .

1 mark

- b. Find and classify the coordinates of the points of inflection on the graph of f .

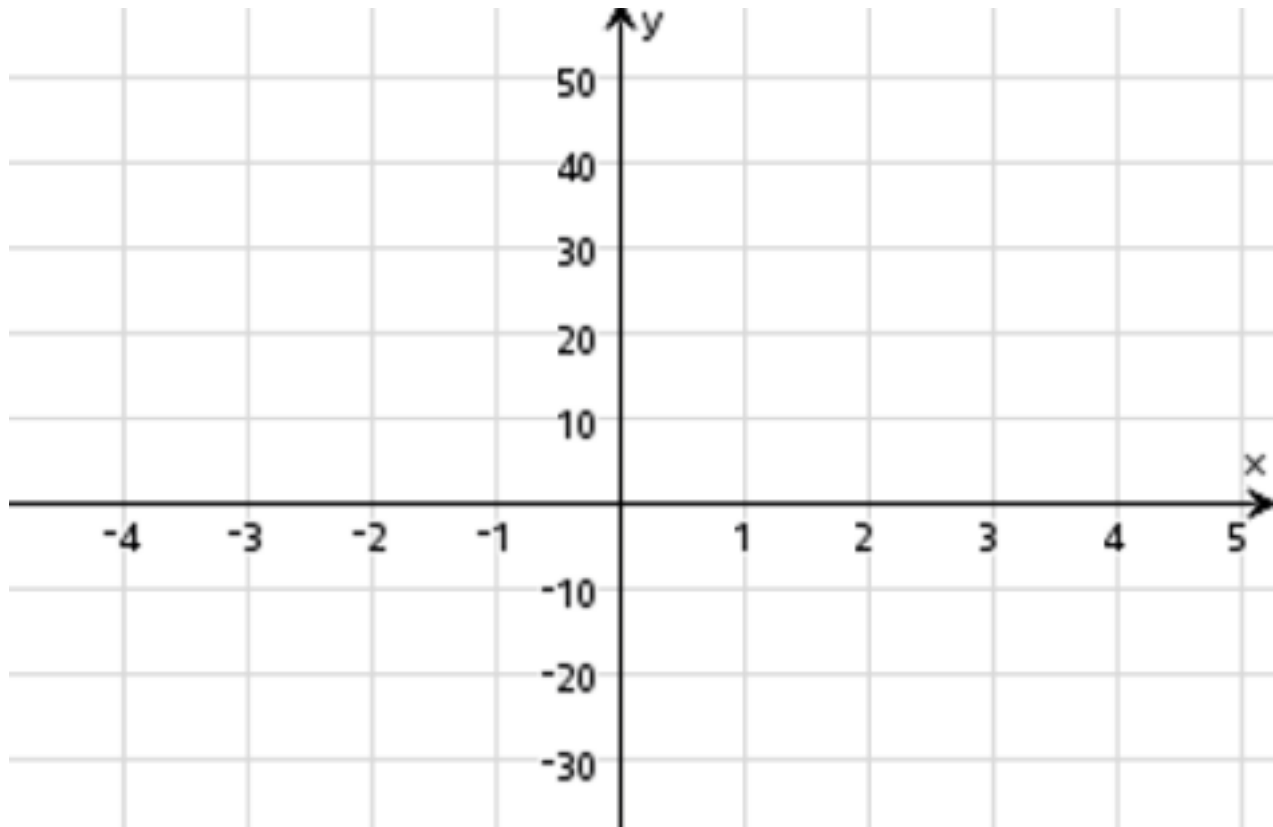
1 mark

- c. Let $g(x)$ be the equation of the tangent to the graph of $f(x)$ at the point, where $x = 2$.
State the rule for $g(x)$.

1 mark

- d. Sketch the graphs of both $f(x)$ and $g(x)$ on the axes below, clearly labelling all stationary points and points of inflexion with their exact values and x -intercepts correct to two decimal places.

2 marks



- e. The area A bounded by the graphs of $f(x)$ and $g(x)$ can be expressed as

$$A = \int_{-b}^b (x+b)(b-x)^n dx.$$

- i. State the values of b and n .

2 marks

ii. State the value of A .

1 mark

f. Find the value(s) of c , where $c > 1$, if the average rate of change of the function f over $1 \leq x \leq c$ is equal to -12 .

1 mark

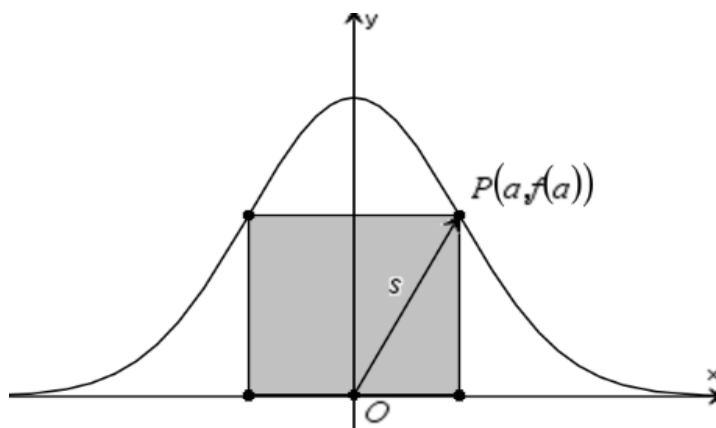
g. Let $h: R \rightarrow R$, $h(x) = f(x) + k$
For what values of k , does the graph of h not cross the x -axis?

1 mark

Question 2 (8 marks)

The diagram shows part of the graph of the function $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = e^{-x^2}$.

The point $P(a, f(a))$ where $a > 0$ lies on the graph of the function f .



a. A rectangle is formed with two points on the x -axis and two points on the graph of the function f , one is at the point P as shown.

i. Find the value of a for which the area of the rectangle is a maximum.

1 mark

ii. Find this maximum area.

1 mark

iii. Jenny stated that the value of a for which the maximum area of the rectangle occurs is also one of the points of inflection on the graph of the function f .
Comment on Jenny's assertion.

1 mark

b. When the point P is the closest distance s on the graph of f to the origin O .

i. Determine the value of a .

1 mark

ii. Find this minimum distance s .

1 mark

c. Describe in words, a sequence of transformations that takes the graph of the function f , to the graph of a normal distribution curve with a mean of μ , where $\mu > 0$ and a standard deviation of σ .

3 marks

Question 3 (16 marks)

Of the new cars sold in Victoria in 2023, they can be classified according to their fuel type as Electric, Hybrid, Petrol or Diesel. The table below shows the percentage of each type of car sold in Victoria in 2023.

Fuel type	Electric	Hybrid	Petrol	Diesel
Percentage	7	16	46	31

- a. i. A random sample of 50 new cars sold in Victoria in 2023 was selected, find the probability that more than 50% used Petrol only. Give your answer correct to four decimal places.

1 mark

- ii. A random sample of n Electric cars were sold in Victoria in 2023. Find the smallest value of n , if the probability of at least two of the n Electric cars exceeds 0.3.

1 mark

- b. For new cars sold in Victoria in 2023, it is known that $b\%$ of the electric or hybrid cars were black, while 40% of the petrol or diesel cars were black. A new car sold in 2023 is selected at random and was found to be non-black, the probability that it is an electric or hybrid car is $\frac{23}{177}$. Determine the value of b .

2 marks

c. The battery lifetime for a hybrid car is normally distributed with a mean lifetime of 8 years, and a standard deviation of 8 months.

i. Find the probability that a hybrid car battery lasts more than 10 years, if it known to have lasted at least 9 years. Give your answer correct to four decimal places.

2 marks

ii. What is the length of time in years, correct to two decimal places, before which 80% of hybrid car batteries need to be replaced.

1 mark

iii. Of a random sample of 10 hybrid cars, find the probability that more than two have a battery that lasts longer than 9 years. Give your answer correct to four decimal places.

2 marks

- d.** It is known that the proportion of white cars in Victoria is 34% and the proportion of black cars in Victoria is 21%. In a car park W it is found that the lower bound for a 95% confidence interval for the proportion of white cars, is equal to the upper bound for a 99% confidence interval for the proportion of black cars in another car park B. Determine the number of white cars in car park W, if there are three times the number of black cars in car park B as the number of white cars in car park W.

3 marks

- e.** When trading in or purchasing another car, it is found that the length of time that Australians keep a car is normally distributed. It is found that 86% of Australians have a car for more than 7 years, while 5% keep a car for less than 5 years. Determine the mean and standard deviation for the length of time that Australians keep a car for, giving both answers in years, correct to one decimal place.

2 marks

The time T in hours to service a car depends upon its engine type and age. The time can vary,

and follows a probability density function given by $f(t) = \begin{cases} b(2t-1) & \text{for } \frac{1}{2} \leq t \leq 1 \\ \frac{b}{\left(t - \frac{1}{2}\right)^2} & \text{for } 1 < t \leq 4 \\ 0 & \text{elsewhere} \end{cases}$

- f. Determine the probability that the time taken to service a car lies within two standard deviations of the mean. Give your answer correct to four decimal places.

2 marks

Question 4 (14 marks)

Consider the function $f : [-\pi, \pi] \rightarrow \mathbb{R}$, $f(x) = x^2 \cos(x)$

- a. Show that the function $f(x)$ is an even function, and describe in words how this relates to the graph of $y = f(x)$.

2 marks

- b. Apply Newton's method with a starting value of $x_0 = 0.75$ to find the approximate x value of the positive x value of the turning point, on the graph of $y = f(x)$, write down the estimates x_1 and x_2 in the table below, correct to three decimal places.

2 marks

x_0	0.75
x_1	
x_2	

- c. i. Let a be a real number, where $0 < a < \pi$. Find in terms of a , the equation of the tangent to f at the point $(a, f(a))$.

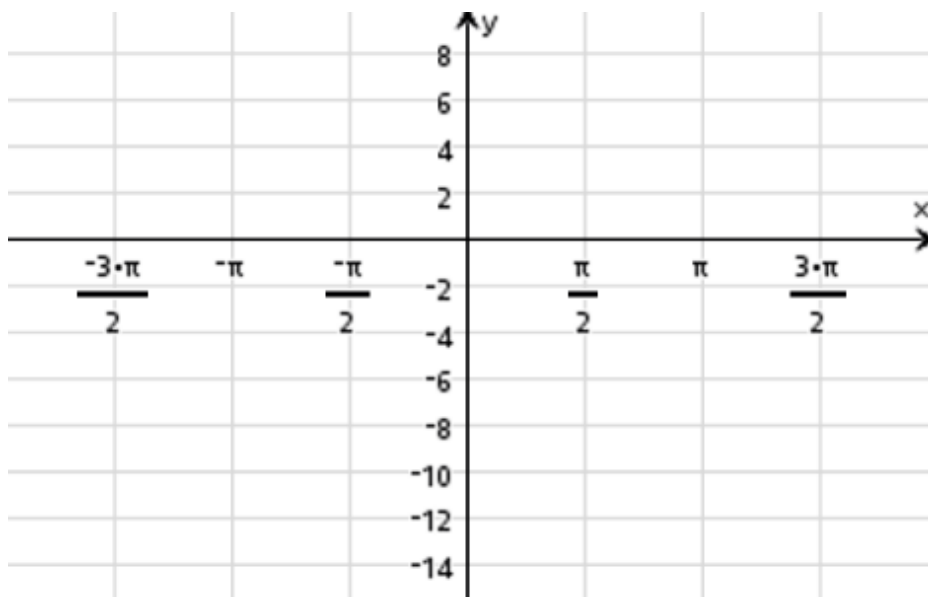
1 mark

- ii. Hence find the equation of the tangent to f that crosses the x -axis at $x = \pi$, give your answers correct to three decimal places.

2 marks

- d. Sketch the graph of $y = f(x)$ on the axes below, labelling all turning points correct to two decimal places and the endpoints with their exact values.

3 marks



- e. Determine the values of x for which the function f is strictly increasing.
Give your answer correct to two decimal places.

1 mark

- f. Find the average value of the function $f(x)$.

1 mark

- g. Consider now the graph of $f_n : [-\pi, \pi] \rightarrow \mathbb{R}$, $f_n(x) = x^n \cos(x)$ where $n \in \mathbb{N}$.
Classify the point at the origin and the values of n for which these occur.

2 marks

Question 5 (12 marks)

Consider the function $f : \left[\frac{3}{2}, \infty\right) \rightarrow R$, $f(x) = \sqrt{4x^2 - 9}$

- a. Find f^{-1} , the inverse function f , stating the domain.

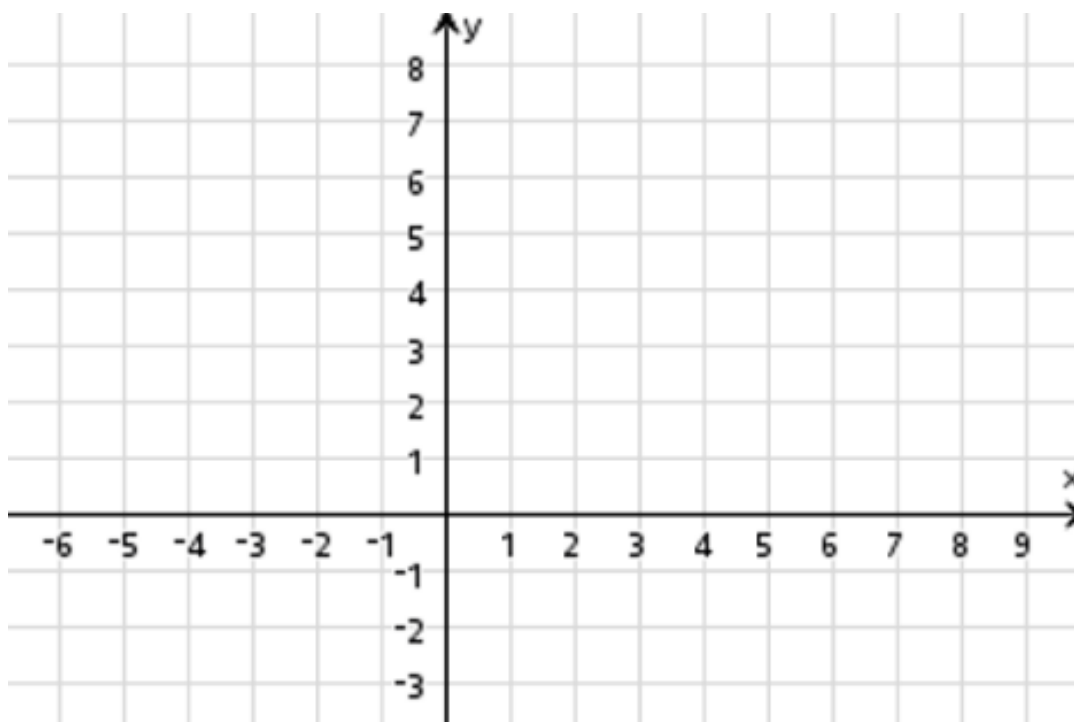
1 mark

- b. Find the coordinates of the point P which is the intersection between the graphs of $y = f(x)$ and $y = f^{-1}(x)$.

1 mark

- c. Sketch the graphs of $y = f(x)$, $y = f^{-1}(x)$ and the graph of $y = x$ on the diagram below, labelling axial intercepts.

2 marks



- d. i.** Write down in terms of definite integrals, the area A enclosed between the graphs of $y = f(x)$ and $y = f^{-1}(x)$ and the line $y = -x + 3(\sqrt{3} + 1)$, and shade this area on the diagram above.

2 marks

- ii.** Evaluate the area A , giving your answer correct to three decimal places.

1 mark

- e.** Find the angle between the tangents to the curves $y = f(x)$ and $y = f^{-1}(x)$ at the point P .
Give your answer in degrees correct to one decimal place.

2 marks

Consider now the one-to-one increasing function $g : D \rightarrow R$, $g(x) = \sqrt{kx^2 - 9}$ where $k \in R$.

- f. Find values of k , when the graphs of $y = g(x)$ and $y = g^{-1}(x)$ do not intersect.

1 mark

- g. The two curves $y = g(x)$ and $y = g^{-1}(x)$ intersect at $x = c$ where $c > 1$. If the angle between the tangents at $x = c$ is $\tan^{-1}\left(\frac{12}{5}\right)$, determine the value of k .

2 marks

END OF SECTION B

**End of question and answer book for the
2024 Kilbaha VCE Mathematical Methods Trial Examination 2**

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MATHEMATICAL METHODS

Written examination 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of triangle	$\frac{1}{2}bc \sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$		
$\frac{d}{dx}((ax+b)^n) = na(ax+b)^{n-1}$	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$		
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$		
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e(x) + c, x > 0$		
$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$		
$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$	$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$		
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$	Newton's method	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
trapezium rule approximation	$Area \approx \frac{x_n - x_0}{2n} [f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-2}) + 2f(x_{n-1}) + f(x_n)]$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A B) = \frac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = E(X)$	variance	$\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$
binomial coefficient	$\binom{n}{x} = \frac{n!}{x!(n-x)!}$		

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
binomial	$\Pr(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$	$\mu = np$	$\sigma^2 = np(1-p)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$

Sample proportions

$\hat{P} = \frac{X}{n}$	mean	$E(\hat{P}) = p$
standard deviation	approximate confidence interval	$\left(\hat{p} - z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right)$

END OF FORMULA SHEET

ANSWER SHEET

STUDENT NUMBER

Figures
Words

Letter

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SIGNATURE _____

SECTION A

1	A	B	C	D
2	A	B	C	D
3	A	B	C	D
4	A	B	C	D
5	A	B	C	D
6	A	B	C	D
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15	A	B	C	D
16	A	B	C	D
17	A	B	C	D
18	A	B	C	D
19	A	B	C	D
20	A	B	C	D

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