Neap

VCE Mathematical Methods Units 3&4

Question and Answer Booklet

2024 Trial Examination 2

Reading time: 15 minutes

Writing time: 2 hours

Student’s Name: ______________________

Teacher’s Name: ______________________

Approved materials

- One approved technology (calculator or software)
- One scientific calculator
- One bound reference

Materials supplied

- Question and Answer Booklet of 19 pages
- Formula Sheet
- Multiple-Choice Answer Sheet

Instructions

- Write **your name** and your **teacher’s name** in the spaces above on this page.
- Follow the instructions on your Multiple-Choice Answer Sheet.
- At the end of the examination, place your Multiple-Choice Answer Sheet inside the front cover of this booklet.

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents pages

Section A

Instructions

- Answer **all** questions in pencil on the Multiple-Choice Answer Sheet.
- Choose the response that is **correct** for the question.
- A correct answer scores 1; an incorrect answer scores 0.
- Marks will **not** be deducted for incorrect answers.
- No marks will be given if more than one answer is completed for any question.
- Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1

Let $f(x) = 3x - 1$ and $g(x) = 6x + 2$.

The value of $(f \circ g)(3)$ is

A. 50

B. 55

C. 59

D. 60

Question 2

The range of the parabola $y = mx^2 - 2mx + 4m$, where $m < 0$, is

A. $[3, \infty)$

B. $[3m, \infty)$

C. $(-\infty, 3]$

D. $(-\infty, 3m]$

Question 3

Consider the following system of simultaneous linear equations.

$$mx + 2y = m - 3$$
$$(k + 1)x + ky = 4$$

The system has infinite solutions when

A. $m = \frac{5}{2}$ and $k = 4$

B. $m = 3$ and $k = -\frac{5}{2}$

C. $m = \frac{5}{3}$ and $k = -6$

D. $m = 4$ and $k = \frac{2}{3}$

Question 4

A discrete random variable X has the following probability distribution.

x	0	1	2	3
$\Pr(X = x)$	0.1	0.25	0.35	0.3

The mean of $2X - 3$ is

A. 0.5

B. 0.7

C. 1.55

D. 1.85

Question 5

Given that $h(x) = f(4x + 7)$, the value of $h'(11)$ is equal to

A. $4 \times f'(1)$

B. $4 \times f'(51)$

C. $\frac{1}{4} \times f'(1)$

D. $\frac{1}{4} \times f'(51)$

Question 6

The maximum value of the function $f : R \to R$, $f(x) = (2\cos(x) - 1)^2$ is

A. 2

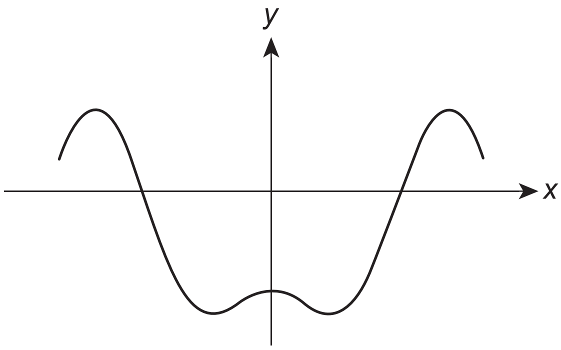
B. $4 - 2\sqrt{3}$

C. $4 + 2\sqrt{3}$

D. 9

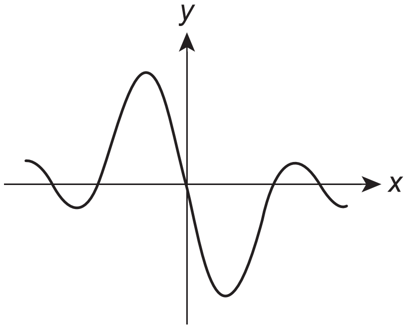
Question 7

Part of the graph of $y = f(x)$ is shown below.

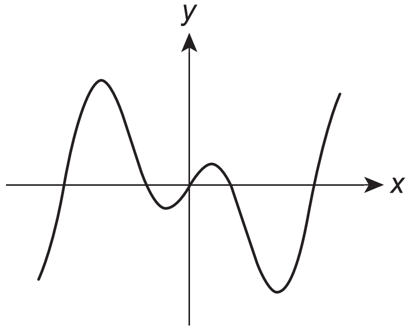


Which one of the following best represents the corresponding part of the graph of $y = f'(x)$?

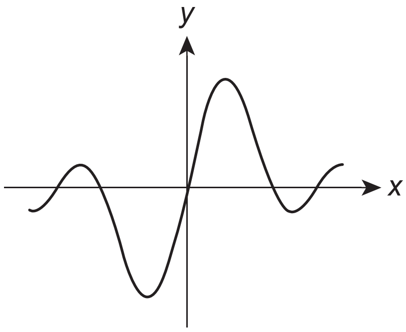
A.



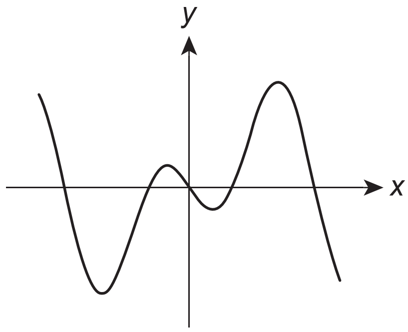
B.



C.



D.



Question 8

Two events, A and B, are independent. It is known that $2\Pr(A) = 3\Pr(B)$ and $\Pr(A \cup B) = 0.34$.

$\Pr(A)$ is closest to

A. 0.1

B. 0.2

C. 0.3

D. 0.4

Question 9

For a random sample of four Australians, $\hat{P}$ is the random variable that represents the proportion of people who have visited a beach in the last year.

If $\Pr(\hat{P} = 1) = \frac{16}{2401}$, then $\Pr(\hat{P} < 0.5)$ is closest to

A. 0.3567

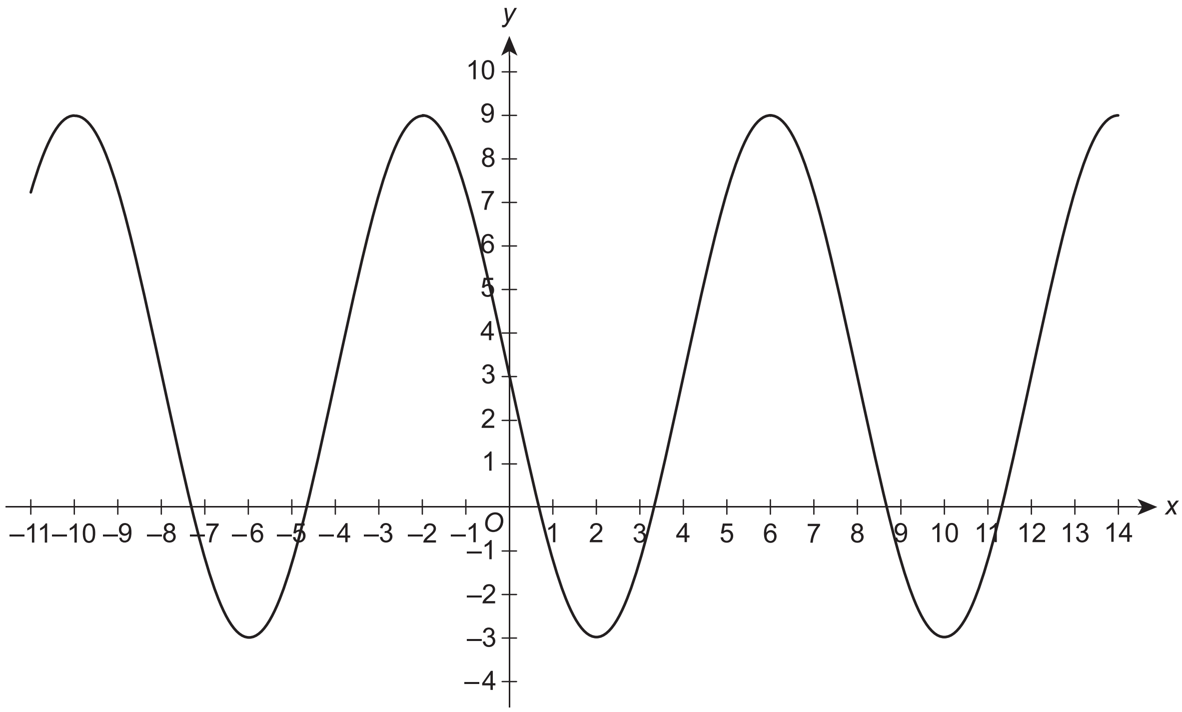
B. 0.6213

C. 0.6768

D. 0.8654

Question 10

The graph of the function *f* is shown below.



Which one of the following is most likely to be function *f* ?

A. $y = 3 - 6\sin\left(\frac{\pi x}{2}\right)$

B. $y = 3 - 6\sin\left(\frac{\pi x}{4}\right)$

C. $y = 3 + 6\cos\left(\frac{\pi x}{2}\right)$

D. $y = 3 + 6\cos\left(\frac{\pi x}{4}\right)$

Question 11

The point (–2, 3) lies on the graph of $y = f(x)$.

If the graph becomes $y = 2f\left(\frac{x}{2}+1\right)-1$, the image of this point is

A. $\left(-6, \frac{5}{2}\right)$

B. $\left(0, \frac{5}{2}\right)$

C. (–6, 5)

D. (0, 5)

Question 12

The continuous random variable X has the following probability density function.

$$f(x) = \begin{cases} \frac{x+k}{10}+1 & 0 \le x \le 2 \\ \frac{1-x}{k} & 2 \le x \le 5 \\ 0 & \text{elsewhere} \end{cases}$$

The value of k is closest to

A. –9.82

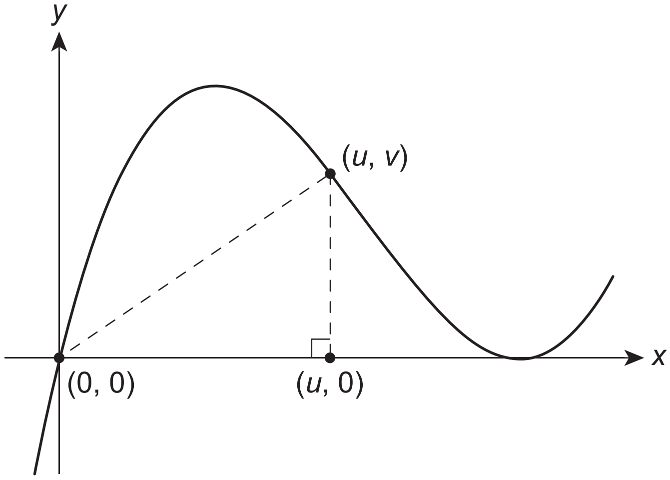
B. –3.82

C. 3.82

D. 9.82

Question 13

A right-angled triangle is formed by joining the points (0, 0), (u, 0) and (u, v). The point (u, v) lies on the graph of $y = x(x-2)^2$, where $0 < u < 2$, as shown below.



What is the maximum area of the triangle?

A. 0.5

B. 0.6

C. 0.7

D. 0.8

Question 14

A box contains four blue marbles, five yellow marbles and two green marbles. Three marbles are drawn at random from the box without replacement.

The probability of drawing marbles of at most two different colours is equal to

A. $\frac{4}{99}$

B. $\frac{240}{1331}$

C. $\frac{25}{33}$

D. $\frac{95}{99}$

Question 15

The random variable X is normally distributed with a mean of 16.

If $\Pr(X > 22) = 0.12$, then $\Pr(X < 15)$ is closest to

A. 0.4224

B. 0.4345

C. 0.4467

D. 0.4578

Question 16

The following algorithm applies Newton's method to approximate the square root of a given number.

```
Inputs :
    num: the number for which square root will be found
    x0: initial estimate
    n: number of iterations
Define approximate square root (num, x0, n):
    If num < 0 Then
        Print "Error: num cannot be negative."
      End If
    x ← x0
    For i = 1 to n
        [                                ]
    End For
    Return x
```

Which one of the following could be appropriate for the missing part of the algorithm?

A. `x ← x + (x * x – num) / (2 * x)`

B. `x ← x – (x * x – num) / (2 * x)`

C. `x ← x – (x * x – num) / 2 * x`

D. `x ← x – (x * x – num / 2) / x`

Question 17

If $\int_1^4 (f(x) + 2x)\,dx = 15$, where f is continuous, which one of the following **must** be true?

A. f is a strictly decreasing function.

B. f is a strictly increasing function.

C. f has a positive y-intercept.

D. f has at least one x-intercept.

Question 18

Consider $f(x) = x^3 + px^2 - px$.

If the graph of f has no stationary points, then

A. $0 < p < 3$

B. $p > 3$

C. $p < -3$

D. $-3 < p < 0$

Question 19

All tangents to the graph of $f : [a, \infty) \to f(x) = \log_e(x - 2)$ have a negative y-intercept.

The minimum possible value of a is closest to

A. 5.92

B. 6.21

C. 6.32

D. 6.67

Question 20

If $\log_{a\sqrt{b}}(c) = x$, which one of the following is correct?

A. $\log_e\left(b^x\right) = 2 + \log_a(c)$

B. $\log_a\left(b^{2x}\right) = \log_c(a)$

C. $\log_a(b) = 1 + \log_c\left(a^{2x}\right)$

D. $\log_c\left(b^x\right) = 2 - \log_c\left(a^{2x}\right)$

End of Section A

Section B

Instructions

- Answer **all** questions in the spaces provided.
- Write your responses in English.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1 (11 marks)

Let $f : R \to R$, $f(x) = (1 - 4x)e^{x}$.

a. State the equation of the derivative function, f'. 1 mark

b. Find the equation of the tangent to the graph of f at $x = 0$. 1 mark

c. Find the range of f. 2 marks

d. **i.** Write a definite integral for the area between the graph of f and the line $y = 1$. Where necessary, give your answer correct to two decimal places. 2 marks

ii. Hence, find the area between the graph of f and the line $y = 1$. Give your answer correct to two decimal places. 1 mark

e. Find the x-coordinates of the points on the graph of f, such that the tangents to the graph of f cross the y-axis at $y = 2.5$. Give your answers correct to two decimal places. 2 marks

f. The graph of $y = f(x)$ is transformed to obtain $y = (ax + b)e^{-2x + 4}$.
State the values of a and b. 2 marks

Question 2 (12 marks)

The waiting time for passengers at a particular bus stop, in minutes, is a continuous random variable, *W*, with the probability density function

$$f(w)=\begin{cases}\dfrac{1}{15}e^{-\frac{w}{15}} & w \geq 0 \\ 0 & \text{elsewhere}\end{cases}.$$

a. Find the mean waiting time. 1 mark

b. On a particular day, 120 passengers use the bus stop.

How many passengers are expected to wait for more than 18 minutes? Give your answer correct to the nearest whole number. 2 marks

c. What is the probability that a passenger waits for less than 20 minutes, given that they wait for at least 18 minutes? Give your answer correct to four decimal places. 2 marks

Once passengers have boarded the bus at the bus stop, the duration of their journey is normally distributed with a mean of 28.3 minutes and a standard deviation of 5.6 minutes.

d. What is the probability that the duration of a passenger's journey is between 10 and 15 minutes? Give your answer correct to four decimal places. 1 mark

e. 20% of passengers complete their journey in less than *T* minutes.

Find *T*, correct to one decimal place. 1 mark

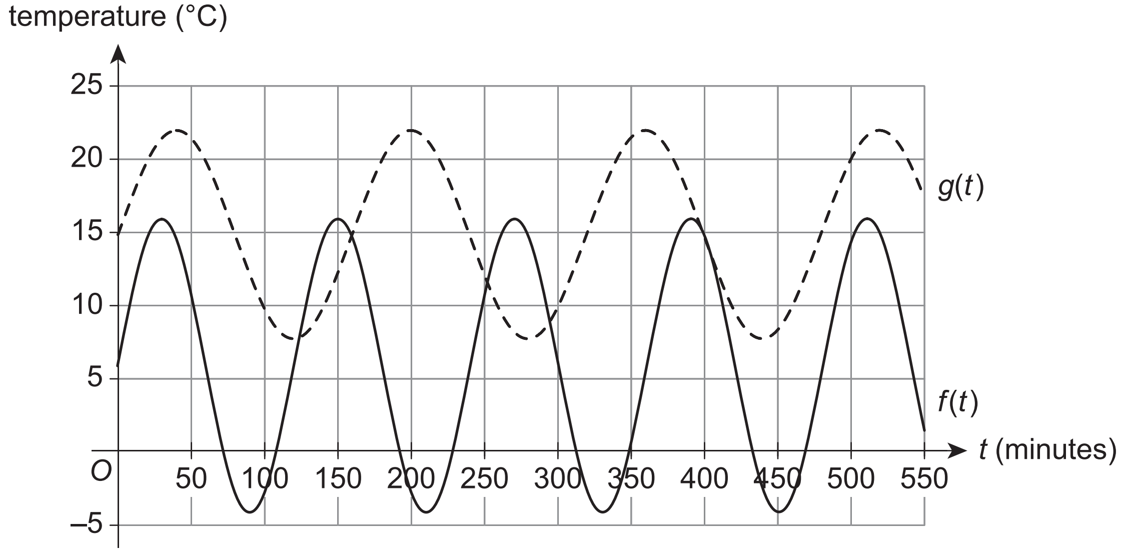
f. Seven passengers board the same bus from the bus stop.

What is the probability that at least three of the passengers travel on the bus for less than 25 minutes? Give your answer correct to four decimal places. 3 marks

g. For a particular sample of passengers, it can be estimated with 90% confidence that between 72% and 84% of passengers will travel on the bus for more than 20 minutes.

Calculate the sample size. 2 marks

Question 3 (14 marks)
Two containers vary in temperature periodically. The graph below shows the temperature of each container over time *t* minutes.



The temperature of the first container can be modelled by the equation $f(t) = 10\sin\left(\frac{\pi t}{60}\right) + 6$.

The temperature of the second container can be modelled by the equation $g(t) = 7\sin\left(\frac{\pi t}{80}\right) + 15$.

a. State the maximum and minimum temperature of the first container. 1 mark

b. State the initial temperature of the second container. 1 mark

c. Let $h(t) = g(t) - f(t)$.
Find the period of $h(t)$. 2 marks

d. Find the maximum difference between the temperatures of the two containers at any one time. Give your answer correct to one decimal place. 3 marks

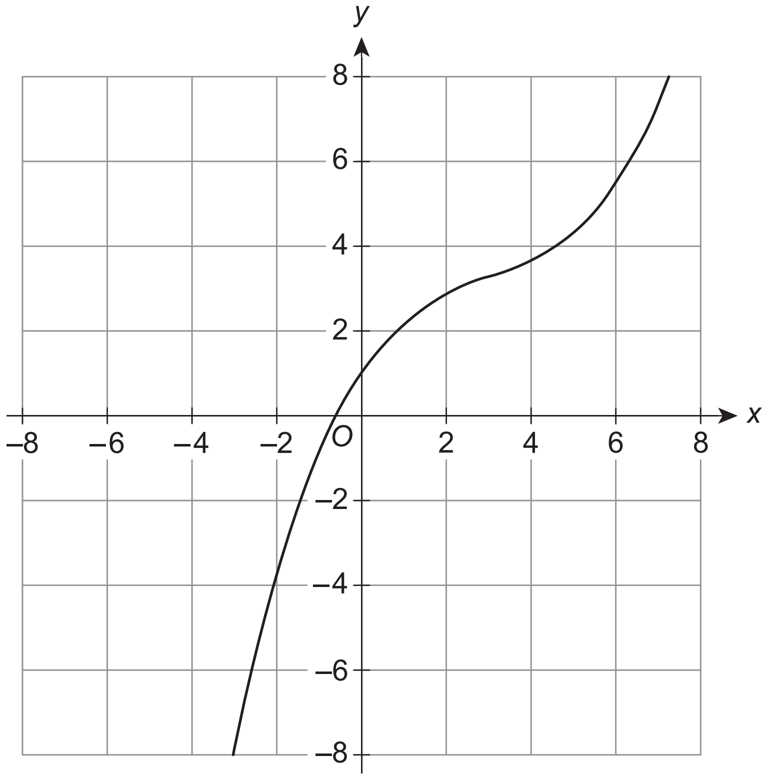
e. Find the average difference between the temperatures of the two containers during the first hour. Give your answer correct to one decimal place. 2 marks

f. At what time will the temperatures of both containers first decrease at the same rate? Give your answer correct to one decimal place. 2 marks

g. State a sequence of transformations that maps the graph of $y = f(t)$ to the graph of $y = g(t)$. 3 marks

Question 4 (11 marks)

Let $f(x) = \frac{1}{24}x^3 - \frac{3}{8}x^2 + \frac{3}{2}x + 1$. The graph of $y = f(x)$ is shown below.



a. Show that the curve $y = f(x)$ crosses the x-axis between $x = -1$ and $x = 0$. 1 mark

b. An estimate of one of the roots for the equation $f(x) = 0$ is $x_0 = -0.5$.

Use two iterations of Newton's method to find a more accurate estimate for this root. Give your answer correct to four decimal places. 3 marks

c. Show that f is strictly increasing for all values of x. 1 mark

d. Explain why $f(x) = 0$ has only one root. 1 mark

e. On the graph on page 16, sketch the graph of $y = f^{-1}(x)$. 2 marks

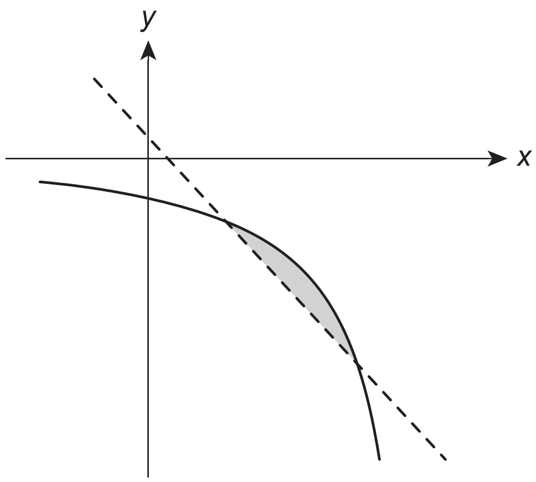
f. Find the area enclosed by f and f^{-1}. Give your answer correct to one decimal place. 3 marks

Question 5 (12 marks)

Let $f: R \setminus \{4\} \rightarrow R, f(x) = \dfrac{ax+2}{x-4}$.

a. Find the equation and domain of the inverse function, f^{-1}, in terms of a. 2 marks

Parts of the graphs of $y = f(x)$ and $y = -x + a$ are shown below.



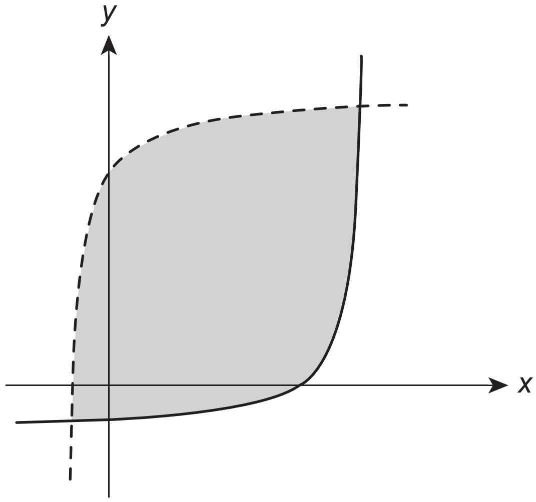
Let the area enclosed by $y = f(x)$ and $y = -x + a$ be S_1.

b. Find the values of a such that $S_1 > 0$. 2 marks

c. Let $a = 0$.

Find S_1. 3 marks

Parts of the graphs of $y = f(x)$ and $y = f^{-1}(x)$ are shown below.



Let the area enclosed by f and f^{-1} be S_2.

d. Find the values of a such that $S_2 > 0$. 3 marks

e. Show that $S_2 < (4 - a)^2$. 2 marks

End of examination questions