Neap

VCE Mathematical Methods Units 3&4

Question and Answer Booklet

2024 Trial Examination 1

Reading time: 15 minutes

Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

Materials supplied
- Question and Answer Booklet of 11 pages
- Formula Sheet

Instructions
- Write your responses in English.
- Write **your name** and your **teacher's name** in the spaces above on this page.

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents pages

Instructions

- Answer **all** questions in the spaces provided.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1 (3 marks)

a. Let $y = (5x^2 - x)^3$.

Find $\dfrac{dy}{dx}$.   1 mark

b. Let $f(x) = e^x \sin(2x)$.

Evaluate $f'\left(\dfrac{\pi}{4}\right)$.   2 marks

Question 2 (2 marks)

Let $f'(x) = \dfrac{4}{x}$, where $x > 0$.

Given that $f(1) = 2$, find $f(x)$.

Question 3 (6 marks)

Let $f : R \setminus \{1\} \to R, f(x) = \dfrac{8}{(x-1)^2} - 2.$

a.   On the axes below, sketch the graph of f. Label the axis intercepts with their
coordinates and any asymptotes with their equations.   4 marks

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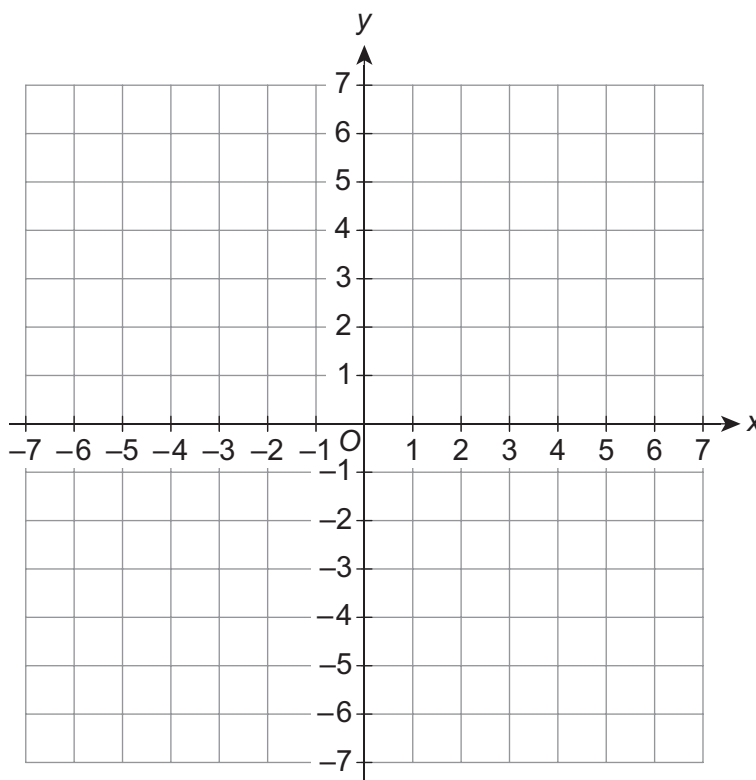
b.   Find the area enclosed by the graph of *f*, the lines *x* = 5 and *x* = 7, and the *x*-axis.   2 marks

Question 4 (3 marks)

A researcher wants to estimate the proportion of people who prefer shopping online rather than in store. She surveys a random sample of 400 people and asks them about their shopping preferences. Out of the sample, 80 people state that they prefer shopping online.

Using $z = 2$, approximate a 95% confidence interval for the proportion of people who prefer shopping online.

Question 5 (3 marks)

Solve $2\cos(2x) + 1 = 0$ for $x \in R$.

Question 6 (6 marks)

Let $f(x) = 3\sin(x) + 1$.

a. State the domain and range of f.   1 mark

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b. Calculate the average rate of change of f between $x = -\dfrac{\pi}{3}$ and $x = \dfrac{\pi}{2}$.   2 marks

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c. Calculate the average value of f over the interval $-\dfrac{\pi}{3} \le x \le \dfrac{\pi}{2}$.   3 marks

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Question 7 (5 marks)

A local bakery sells boxes of muffins. Each box contains 20 muffins, each of which is one of four types:

- blueberries and nuts
- blueberries **only**
- nuts **only**
- neither blueberries nor nuts

It is known that:

- 10 muffins in the box contain nuts
- 8 muffins in the box do not contain blueberries
- 4 muffins in the box contain both blueberries and nuts.

a. Two muffins are chosen at random from one box.

Find the probability that both muffins contain blueberries **only**.   2 marks

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b. One muffin is chosen at random from each of three boxes. It is known that exactly two of the selected muffins contain blueberries.

Find the probability that none of the selected muffins contain nuts.   3 marks

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Question 8 (6 marks)

Let $f : (-3, 0) \rightarrow R$, where $f(x) = \log_e \left(x^3 + 3x^2 \right)$ and $g : R \rightarrow R$, where $g(x) = e^x$.

a. Find the coordinates of the stationary point of f and state its nature. 3 marks

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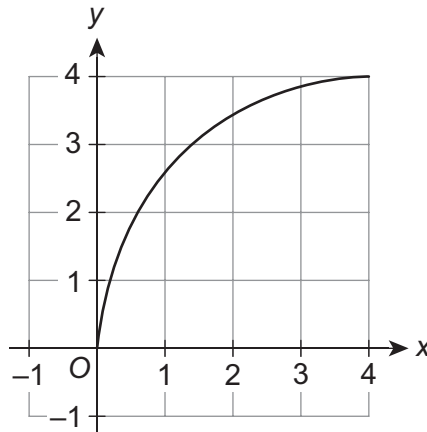
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Let $h(x) = g(f(x))$.

b. Simplify the equation of h. 1 mark

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c. State the domain and range of h. 2 marks

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Question 9 (6 marks)

Consider $f : [0, 4] \to R$, $f(x) = \sqrt{8x - x^2}$. Part of the graph of $y = f(x)$ is shown below.

a. Determine the equation of the inverse function, f^{-1}.   2 marks

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b. Show that the graph of $y = f(x)$ is a circle.   1 mark

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c. Use two trapeziums of equal width to approximate the area between the curve $y = f(x)$, the lines $x = 0$ and $x = 4$, and the x-axis.   **2 marks**

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d. Use your answer from **part c.** to approximate the value of π.   **1 mark**

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End of examination questions