Neap

VCE Mathematical Methods Units 3&4

Suggested Solutions

2024 Trial Examination 1

Question 1 (3 marks)

a. $\frac{dy}{dx} = 3\left(5x^2 - x\right)^2 (10x - 1)$ A1

OR

$\frac{dy}{dx} = 3x^2(5x - 1)^2(10x - 1)$ A1

b. $f'(x) = e^x \times \sin(2x) + e^x \times 2\cos(2x)$

$= e^x \left(\sin(2x) + 2\cos(2x)\right)$ A1

$f'\left(\frac{\pi}{4}\right) = e^{\frac{\pi}{4}}\left(\sin\left(\frac{\pi}{2}\right) + 2\cos\left(\frac{\pi}{2}\right)\right)$

$= e^{\frac{\pi}{4}}(1 + 0)$

$= e^{\frac{\pi}{4}}$ A1

Question 2 (2 marks)

$f(x) = \int \frac{4}{x}\,dx$

$= 4\log_e(x) + c$ A1

$f(1) = 2$

$4\log_e(1) + c = 2$

$0 + c = 2$

$c = 2$

$\therefore\ f(x) = 4\log_e(x) + 2$ A1

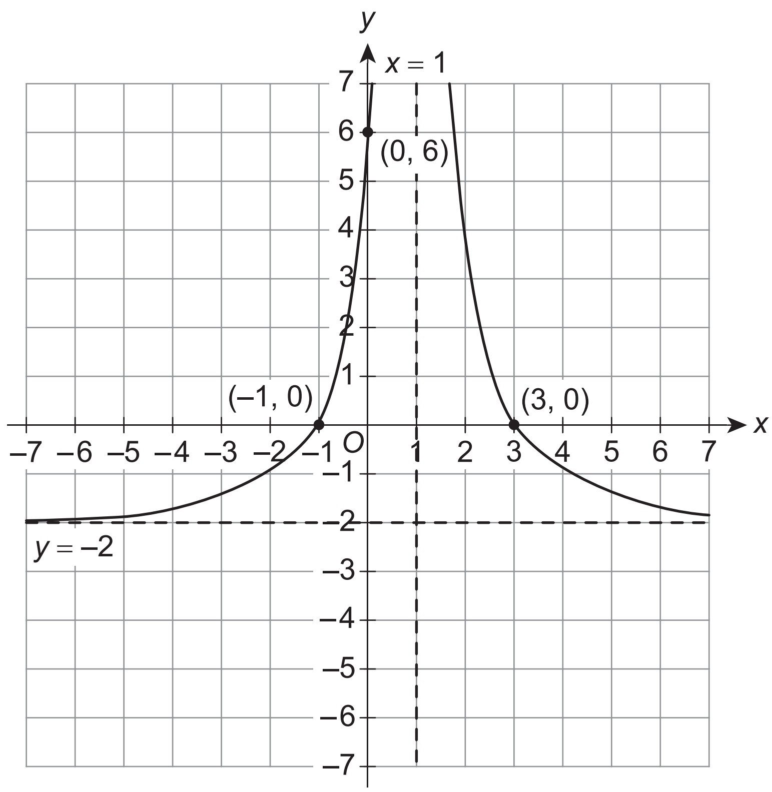
Question 3 (6 marks)

a. When $x = 0$:

$$y = \frac{8}{(0-1)^2} - 2$$
$$= 6$$

When $y = 0$:

$$\frac{8}{(x-1)^2} - 2 = 0$$
$$\frac{8}{(x-1)^2} = 2$$
$$(x-1)^2 = 4$$
$$\left[\begin{array}{l} x-1=2 \\ x-1=-2 \end{array}\right. \Rightarrow \left[\begin{array}{l} x=3 \\ x=-1 \end{array}\right.$$



correct shape A1
correct y-intercept A1
correct x-intercepts A1
correct asymptotes with equations A1

b. $-\int_5^7 \left(\frac{8}{(x-1)^2}-2\right)dx$ A1

$$=-\int_5^7 \left(8(x-1)^{-2}-2\right)dx$$

$$=-\left[-8(x-1)^{-1}-2x\right]_5^7$$

$$=\left(\frac{8}{6}+14\right)-(2+10)$$

$$=\frac{20}{6}$$

$$=\frac{10}{3}$$ A1

Question 4 (3 marks)

$$\hat{p}=\frac{80}{400}$$

$$=\frac{1}{5}$$ A1

$$95\%\ \text{CI}=\left(\hat{p}-x\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p}+x\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$$

$$=\left(\frac{1}{5}-2\sqrt{\frac{\frac{1}{5}\times\frac{4}{5}}{400}}, \frac{1}{5}+2\sqrt{\frac{\frac{1}{5}\times\frac{4}{5}}{400}}\right)$$ M1

$$=\left(\frac{1}{5}-\frac{4}{100}, \frac{1}{5}+\frac{4}{100}\right)$$

$=(0.16, 0.24)$ **OR** $\left(\frac{4}{25}, \frac{6}{25}\right)$ A1

Question 5 (3 marks)

$$\cos(2x)=-\frac{1}{2}$$ M1

$$\left[\begin{array}{l} 2x=\frac{2\pi}{3}+2\pi k \\ 2x=\frac{4\pi}{3}+2\pi k \end{array}\right., k\in Z$$ M1 (if $\frac{2\pi}{3}$ or $\frac{4\pi}{3}$ is obtained)

$\left[\begin{array}{l} x=\frac{\pi}{3}+\pi k \\ x=\frac{2\pi}{3}+\pi k \end{array}\right.$ **OR** $\left[\begin{array}{l} x=\frac{\pi}{3}+\pi k \\ x=-\frac{\pi}{3}+\pi k \end{array}\right., k\in Z$ A1

Question 6 (6 marks)

a. domain = R

range = [–2, 4] A1

b. $$\frac{f\left(\frac{\pi}{2}\right)-f\left(-\frac{\pi}{3}\right)}{\frac{\pi}{2}-\left(-\frac{\pi}{3}\right)}=\frac{(3+1)-\left(-\frac{3\sqrt{3}}{2}+1\right)}{\frac{5\pi}{6}}$$ M1

$$=\frac{\frac{6+3\sqrt{3}}{2}}{\frac{5\pi}{6}}$$

$$=\frac{18+9\sqrt{3}}{5\pi}$$ A1

c. $$\frac{1}{\frac{\pi}{2}-\left(-\frac{\pi}{3}\right)}\int_{-\frac{\pi}{3}}^{\frac{\pi}{2}}(3\sin(x)+1)\,dx$$ M1

$$=\frac{1}{\frac{5\pi}{6}}\left[-3\cos(x)+x\right]_{-\frac{\pi}{3}}^{\frac{\pi}{2}}$$ A1

$$=\frac{6}{5\pi}\left[\left(0+\frac{\pi}{2}\right)-\left(-\frac{3}{2}-\frac{\pi}{3}\right)\right]$$

$$=\frac{6}{5\pi}\left(\frac{5\pi}{6}+\frac{3}{2}\right)$$

$$=\frac{5\pi+9}{5\pi}$$ A1

Question 7 (5 marks)

a.

	With blueberries (B)	Without blueberries (B')	Total
With nuts (N)	4 *(given)*	6	10 *(given)*
Without nuts (N')	8	2	10
Total	12	8 *(given)*	20 *(given)*

valid attempt to find the number of muffins that contain blueberries only M1

$$\Pr(\text{both blueberries only})=\frac{8}{20}\times\frac{7}{19}$$

$$=\frac{14}{95}$$ A1

b. Pr(3 without nuts | 2 blueberries)

$$= \frac{\Pr(\text{2 blueberries only and 1 neither blueberries nor nuts})}{\Pr(\text{2 blueberries and 1 without blueberries})}$$ M1

$$= \frac{\frac{8}{20} \times \frac{8}{20} \times \frac{2}{20} \times 3}{\frac{12}{20} \times \frac{12}{20} \times \frac{8}{20} \times 3}$$ M1

$$= \frac{1}{9}$$ A1

Note: Consequential on answer to ***Question 7a.***

Question 8 (6 marks)

a. $f'(x) = \frac{3x^2 + 6x}{x^3 + 3x^2}$ A1

$$= \frac{3x(x+2)}{x^2(x+3)}$$

$$= \frac{3(x+2)}{x(x+3)}$$

$f'(x) = 0$

$x = -2$ A1

$$\begin{cases} f'(-2^-) > 0 \\ f'(-2^+) < 0 \end{cases}$$

Therefore, the coordinates of the stationary point are $(-2, \log_e(4))$, and the point is a maximum. A1

b. $h(x) = g(f(x))$

$$= e^{\log_e(x^3 + 3x^2)}$$

$$= x^3 + 3x^2$$ A1

c. $D_h = D_f = (-3, 0)$ A1

$R_f = (-\infty, \log_e(4)]$

$e^{-\infty} \to 0$

$e^{\log_e(4)} = 4$

$R_h = (0, 4]$ A1

Question 9 (6 marks)

a. Swapping the values of x and y gives:

$$x = \sqrt{8y - y^2}$$

$$x^2 = 8y - y^2$$

$$y^2 - 8y + x^2 = 0$$ M1

Solving for y gives:

$$y_1 = \frac{8 - \sqrt{64 - 4x^2}}{2}$$

$$= 4 - \sqrt{16 - x^2} \leq 4$$

$$y_2 = \frac{8 + \sqrt{64 - 4x^2}}{2}$$

$$= 4 + \sqrt{16 - x^2} \geq 4$$

Considering $D_f = R_{f^{-1}}$:

$$f^{-1}(x) = 4 - \sqrt{16 - x^2}$$ A1

b.

$$y = \sqrt{8x - x^2}$$

$$y^2 = 8x - x^2$$

$$x^2 - 8x + 16 + y^2 = 16$$

$$(x - 4)^2 + y^2 = 16$$ M1

c.

$$\text{area} = \frac{\frac{4-0}{2}}{2}\left(f(0) + 2f(2) + f(4)\right)$$ M1

$$= 0 + 2\sqrt{12} + 4$$

$$= 4\sqrt{3} + 4$$ A1

d. The value in **part c.** approximates the area of a quarter circle with radius 4.

$$\frac{\pi \times 4^2}{4} \approx 4\sqrt{3} + 4$$

$$\pi \approx \sqrt{3} + 1$$ A1

Note: Consequential on answer to ***Question 9c.***