TSSM
Creating VCE Success

2024 Trial Examination

THIS BOX IS FOR ILLUSTRATIVE PURPOSES ONLY

STUDENT NUMBER | | | | | | | | | Letter

SPECIALIST MATHEMATICS

Written examination 2

Reading time: 15 minutes
Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	5	5	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.
- No calculator is permitted in this examination.

Materials supplied

- Question and answer book of 23 pages.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

SECTION A – Multiple-choice questions

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores zero.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1

"For $x \in N, t$here exists a prime number of the form: $x^2 + 3x + 2$".
This statement:

A. can be proved false using the counterexample: $1^2 + 3 \times 1 + 2 = 6$
B. can be proved true by considering the contrapositive to the statement above
C. can be proved true using proof by induction.
D. can be proved false since: $x^2 + 3x + 2 = (x+1)(x+2)$, establishes that $x^2 + 3x + 2$ has more than 2 factors

Question 2

Consider the statements:
A: n is an even number.
B: n is an odd number.
C: $n^2 + n$ is an even number.
D: $n^2 + n$ is an odd number.
E: n is a natural number.

Which statement **below** is true?

A. $E \Rightarrow C$
B. $D \Rightarrow B$
C. $A \Leftrightarrow C$
D. $E \Rightarrow A$

Question 3

Which statement relating to the function $f(x) = \frac{2x}{\sqrt{x^2-36}} + 2$, is **false**?

A. $x = \pm 6$ are the equations of the vertical asymptotes

B. $y = 2$ is the equation of the horizontal asymptote

C. The implied domain of f is $x \in (-\infty, -6) \cup (6, \infty)$

D. The point $(10,5)$ lies vertically above the graph of $y = f(x)$

Question 4

Consider the algorithm, written in pseudocode:

$a \leftarrow 1$
$b \leftarrow 1$
$while\ b < 100$
$a \leftarrow a + 1$
$b \leftarrow a^3 - b$
$end\ while$
$print\ a, b$

The final set of values for a and b in the printout are:

A. $a = 3,\ b = 20$

B. $a = 3,\ b = 44$

C. $a = 4,\ b = 44$

D. $a = 5,\ b = 81$

Question 5

Consider the graphs of the functions: $y = \sec 3\theta$ and $y = \text{cosec}\, 2\theta$

The number of points of intersection of the two functions over $\theta \in [0, 4\pi]$ is:

A. 4

B. 6

C. 8

D. 10

TURN OVER

Question 6
The implied domain of $y = cos^{-1}((1-2x) - sin^{-1}(\frac{1}{2x})$ is:

A. $x \in [-\frac{1}{2}, 1]$
B. $x \in [\frac{1}{2}, 1]$
C. $x \in (\frac{1}{2}, 1]$
D. $x \in (-\frac{1}{2}, 1]$

Question 7
Given $\cot a = b,\ a \in \left(0, \frac{\pi}{2}\right),\ \sin 2a =$

A. $\frac{b}{1+b^2}$
B. $\frac{2b}{1+b^2}$
C. $\frac{2b}{1-b^2}$
D. $\frac{b}{1-b^2}$

Question 8
Given $z_1 = 2cis\left(-\frac{\pi}{6}\right)$ and $z_2 = 4cis\left(\frac{\pi}{3}\right)$
$\bar{z}_1\sqrt{z_2} =$

A. $2\sqrt{3} + 2\sqrt{3}i$
B. $2 + 2\sqrt{3}i$
C. $2\sqrt{3} + 2i$
D. $2 - 2\sqrt{3}i$

Question 9

Equilateral triangle ABC has $\overrightarrow{AB} = 2\underset{\sim}{i}$ and $\overrightarrow{BC} = -\underset{\sim}{i} + m\underset{\sim}{j}, m > 0$

Find the value of m hence find the scalar product $\overrightarrow{AB}.\overrightarrow{BC}$

A. $m = \sqrt{2},\ \overrightarrow{AB}.\overrightarrow{BC} = 2$

B. $m = -\sqrt{2},\ \overrightarrow{AB}.\overrightarrow{BC} = -2$

C. $m = \sqrt{3},\ \overrightarrow{AB}.\overrightarrow{BC} = -2$

D. $m = -\sqrt{3},\ \overrightarrow{AB}.\overrightarrow{BC} = -2$

Question 10

Given $\underset{\sim}{a} = \underset{\sim}{i} + \underset{\sim}{k}$ and $\underset{\sim}{b} = -2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$, the vector resolute of $\underset{\sim}{a}$ perpendicular to $\underset{\sim}{b}$ is $\frac{\underset{\sim}{c}}{9}$

$\underset{\sim}{c} =$

A. $7\underset{\sim}{i} - 2\underset{\sim}{j} + 10\underset{\sim}{k}$

B. $7\underset{\sim}{i} + 2\underset{\sim}{j} + 10\underset{\sim}{k}$

C. $-7\underset{\sim}{i} - 2\underset{\sim}{j} + 10\underset{\sim}{k}$

D. $7\underset{\sim}{i} - 2\underset{\sim}{j} - 10\underset{\sim}{k}$

Question 11

Find the vector equation of the line of intersection of the planes with vector equations:

$\Pi_1 = \underset{\sim}{r}\,.\left(-\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}\right) = 2$ and $\Pi_2 = \underset{\sim}{r}\,.\left(2\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}\right) = 4$

A. $\underset{\sim}{r} = 6\underset{\sim}{i} - 4\underset{\sim}{k} + \lambda\left(\underset{\sim}{j} + 2\underset{\sim}{k}\right), \lambda \in R$

B. $\underset{\sim}{r} = 6\underset{\sim}{i} - 4\underset{\sim}{k} + \lambda\left(\underset{\sim}{j} - 2\underset{\sim}{k}\right), \lambda \in R$

C. $\underset{\sim}{r} = 6\underset{\sim}{i} + \underset{\sim}{j} - 4\underset{\sim}{k} + \lambda\left(\underset{\sim}{j} + \frac{1}{2}\underset{\sim}{k}\right), \lambda \in R$

D. $\underset{\sim}{r} = 6\underset{\sim}{i} - 4\underset{\sim}{k} + \lambda\left(\underset{\sim}{j} + \frac{1}{2}\underset{\sim}{k}\right), \lambda \in R$

Question 12

Integration by parts can be used to re-express $\int u\frac{dv}{dx}dx$ as $uv-\int v\frac{du}{dx}dx$ for appropriate substitutions for u and v. This technique can be used to find:

A. $\int(\tan^{-1}x)dx$ where $u=\tan^{-1}x$ and $\frac{dv}{dx}=1$

B. $\int(\tan^{-1}x)dx$ where $u=\tan x$ and $\frac{dv}{dx}=1$

C. $\int(x\tan^{-1}x)dx$ where $u=x$ and $\frac{dv}{dx}=\tan^{-1}x$

D. $\int(x\tan x)dx$ where $u=x$ and $\frac{dv}{dx}=\tan x$

Question 13

A particle, moving in a straight line with an initial velocity of $6\ ms^{-1}$ accelerates at $(0.5v-1)ms^{-2}$. Find, correct to two decimal places, the displacement of the particle after $2\ s$.

A. $17.70\ m$

B. $17.75\ m$

C. $17.80\ m$

D. $17.85\ m$

Question 14

The acceleration of a particle oscillating in a straight line with an initial velocity of $12\ cms^{-1}$ is given by the rule: $a=-36x$ where x is the particle's position in centimetres at time t seconds. Given the particle is initially at $x=0$, the magnitude of its acceleration after $2.5\ s$ is closest to:

A. $29\ cms^{-2}$

B. $33\ cms^{-2}$

C. $41\ cms^{-2}$

D. $47\ cms^{-2}$

Question 15

Two independent, normally distributed random variables X and Y are such that:

$E(X) = 24, Var(X) = 1, E(Y) = 30, Var(Y) = 2.$

Given $C = 4X - 3Y$, find correct to 2 decimal places, the probability that a random observation of C will be negative.is:

A. 0.09

B. 0.11

C. 0.13

D. 0.15

Question 16

Based on a sample of n days, the length of time Claire practises on the piano each afternoon is normally distributed with a mean of $\bar{x}\ minutes$ and a standard deviation of $5\ minutes.$

Find the smallest value of n in order that Claire's father can be 90% certain that the actual mean time Claire spends practising each day is within 2 minutes of the sample mean.

A. 15

B. 17

C. 19

D. 21

Question 17

Which statement relating to hypothesis testing for the mean of a population is **false**?

A. The null hypothesis H_0, proposes that any difference between the sample mean and the population mean is due to random variation.

B. The alternative hypothesis H_1, proposes that the difference between the sample mean and the population mean is too significant to be purely due to random variation.

C. The chance that sample statistic is less extreme than the one observed is known the $p - value.$

D. A $p - value$ greater than 0.05 suggests little or no evidence to reject H_0

TURN OVER

Question 18
A particle moves with an acceleration of: $\underset{\sim}{a}(t) = 2\sin 2t\, \underset{\sim}{i} - 4\cos 2t\, \underset{\sim}{j}$

$\underset{\sim}{i}$ represents 1 metre in the $x-$direction and $\underset{\sim}{j}$ represents 1 metre in the $y-$direction.

The initial velocity of the particle is $\left(\underset{\sim}{i} + 2\underset{\sim}{j}\right)\, ms^{-1}$ and the initial position of the particle is $\left(\underset{\sim}{i} - 2\underset{\sim}{j}\right)\, m$. The speed of the particle after π seconds is:

A. $\sqrt{3}\, ms^{-1}$
B. $2\, ms^{-1}$
C. $\sqrt{5}\, ms^{-1}$
D. $\sqrt{6}\, ms^{-1}$

Question 19
The length of the curve defined by the parametric equations:
$x = 4\tan 2t, y = \cos t\, , 0 \le t \le \frac{\pi}{6}$ is closest to:

A. 6.91
B. 6.93
C. 6.95
D. 6.97

Question 20
A projectile is fired from 2 $metres$ above horizontal ground and then lands at an angle of $\theta°$ to ground. Its position is given by: $\underset{\sim}{x}(t) = 20t\underset{\sim}{i} + (-4.9t^2 + 20t + 2)\underset{\sim}{j}$. Let $\underset{\sim}{i}$ be a unit vector of $1\, m$ directly forward and $\underset{\sim}{j}$ be a unit vector of $1\, m$ directly upwards from ground level. Find θ to the nearest degree and minute. (Assume the ground remains horizontal.)

A. $45°40'$
B. $45°50'$
C. $46°00'$
D. $46°20'$

SECTION B – Extended response questions

Instructions for Section B

Answer **all** questions in the spaces provided.

In **all** questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** to scale.

Question 1 (12 marks)

A tank contains $400\ L$ of saltwater with a concentration of salt (S) of $4\ gL^{-1}$. Fresh water is poured into the tank at a rate of $5\ Lmin^{-1}$. The mixture is stirred and removed at a rate of $10\ Lmin^{-1}$.

Let there be S grams of salt in the tank after t minutes.

a. Write a rule for the volume of liquid V litres, in the tank after t minutes.

____

____

1 mark

b. How much salt (in grams) is in the tank at $t = 0$.

____

____

1 mark

TURN OVER

c. Show that the differential equation that describes the rate of change of salt in the tank after t minutes is:

$$dS/dt = \frac{2S}{t-80}$$

2 marks

d. Solve the differential equation to find the amount of salt (S) in the tank after t minutes.

3 marks

e. Find the concentration of salt in the tank when the rate of loss of salt is $30\ g\ min^{-1}$.

3 marks

f. Find $\frac{dV}{dS}$ when the rate of loss of salt is $30\ g\ min^{-1}$.

2 marks

1 + 1 + 2 + 3 + 3 + 2 = 12 marks.

TURN OVER

Question 2 (10 marks)

Consider the two points A and B in three-dimensional space with position vectors:

$\overrightarrow{OA} = \underset{\sim}{a} = -\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$ and $\overrightarrow{OB} = \underset{\sim}{b} = -2\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}$

a. Find $\overrightarrow{AB}$

1 mark

b. Write the vector equation of the line containing the points A and B in the form:
$\underset{\sim}{r}(t) = \underset{\sim}{a} + t\left(\underset{\sim}{b} - \underset{\sim}{a}\right), t \in R$ hence show that the point $C = (-3, 0, 5)$ is on this line.

2 marks

c. Hence, by using point C, find the vector equation of the line parallel to AB passing through the point $D = (1, -2, 1)$.

2 marks

d. The points A and B are in the plane with equation $mx + y + z = n,\ \ m, n \in R$
Find the values of m and n.

2 marks

e. Find the distance of the plane specified in **part d.** from the point $E = (-1, 0, 1)$.

3 marks

1 + 2 + 2 + 2 + 3 = 10 marks.

TURN OVER

Question 3 (11 marks)

ROAR produce car batteries with an average life span that is claimed to be 60 months. A sample of 32 batteries are monitored over an extended period of time. The resulting sample mean life, and standard deviation, that can be assumed to be normally distributed, are 61 and 6 months respectively.

a. Find, correct to one decimal place, the 95% confidence interval for the actual mean life the batteries.

2 marks

b. Find, correct to 2 decimal places, the chance that the actual mean life of ROAR batteries is greater than 63 months.

2 marks

Based on the sample results, management at the company are wondering if their batteries have a mean life of greater than 60 months.

c. Write down an appropriate one-sided null hypothesis H_0 and an alternative hypothesis H_1

1 mark

d. Find, correct to 3 decimal places, the $p-$ value for this test.

2 marks

e. Test at the 5% level of significance, the proposition that the batteries actually have a mean life greater than 60 months.

1 mark

f. ROAR have also been developing a new battery ROARPLUS that they believe is superior to the original ROAR batteries. A sample of n ROARPLUS batteries produce a mean life of 62 months. The company correctly conclude that ROARPLUS has a mean life higher than 60 months at the 1% level of significance. Find the smallest possible value of n. (Assume the standard deviation remains at 6 months.)

3 marks

2 + 2 + 1 + 2 + 1 + 3 = 11 marks

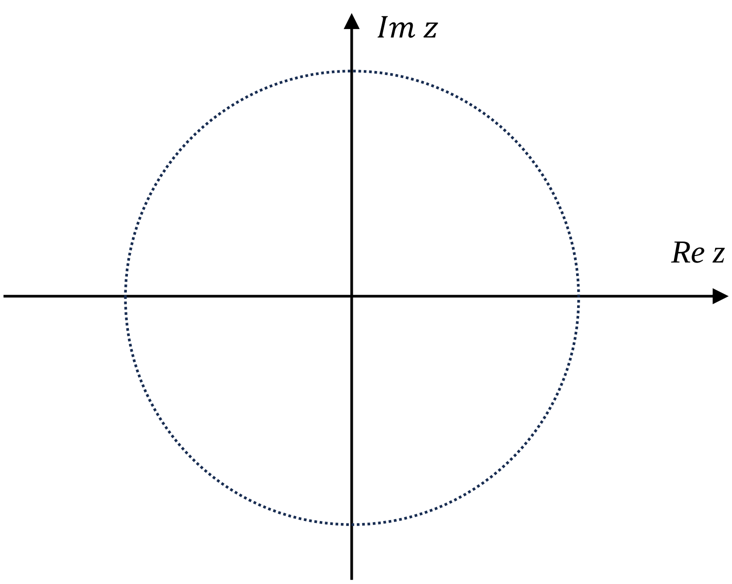
TURN OVER

Question 4 (12 marks)

a. Solve the quartic equation $z^4 = -64,\ z \in C$, writing your four solutions in both polar and cartesian form.

3 marks

b. Show the four solutions from **part a.** on the Argand Plane including the radius of the circle on which they lie.



3 marks

c. Two of the solutions to $z^4 = -64,\ z \in C$, are the roots of a quadratic equation with real coefficients. Find two possible equations.

2 marks

d. Write the equation of the circle illustrated in **part b.** in both cartesian and polar form.

2 marks

TURN OVER

e. The graph of the solutions to $z^4 = -64,\ z \in C$, are now translated 2 units in the positive real direction and 2 units in the negative imaginary direction to now represent a related equation. Identify this quartic equation and its 4 solutions.

2 marks

3 + 3 + 2 + 2 + 2 = 12 marks

Question 5 (15 marks)

Consider the function $f: R \to R$ where $f(x) = \frac{x-4}{x^2+4}$

a. Find the x and y intercept of f.

2 marks

b. Find the exact coordinates of the stationary points on f hence find the implied range of f.

2 marks

c. Find, correct to two decimal places, the steepest positive gradient on f.

2 marks

TURN OVER

d. Use a calculus technique to find the exact value of $\int\left(\frac{x-4}{x^2+4}\right)dx$ (Show working.)

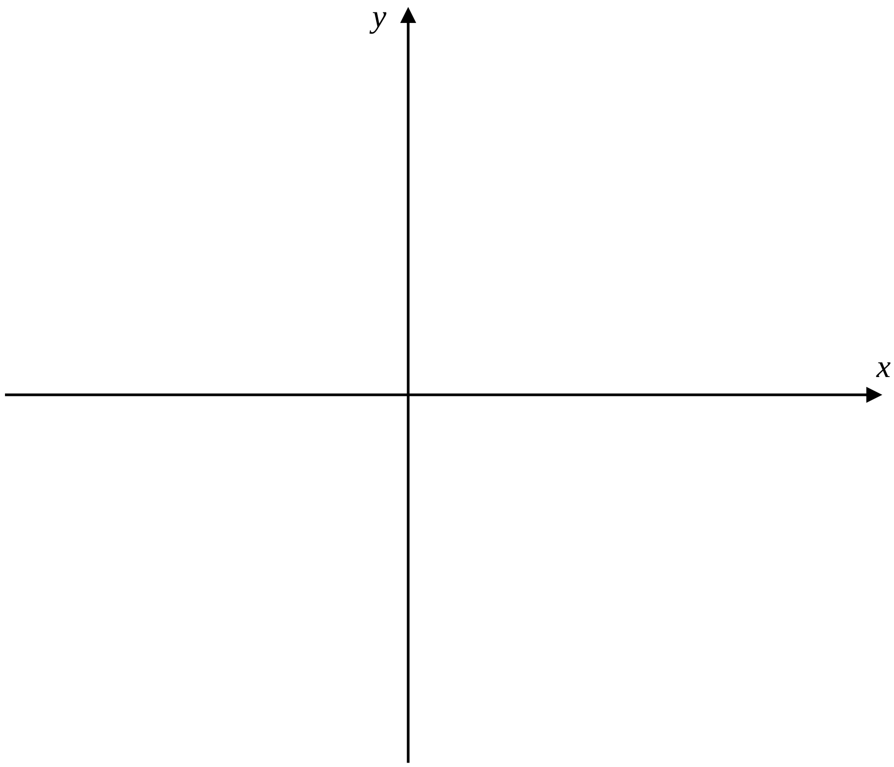
2 marks

e. The area bound by $y = f(x)$ and the straight line $x - 5y = 4$ is of the form:
$log_e(a) + b\tan^{-1}\left(\frac{1}{2}\right) - \pi - c,\ \ a, b, c \in R$
Use your result from **part d.** to evaluate a, b, c.

2 marks

TURN OVER

f. Sketch $f: R \to R$ where $f(x) = \frac{x-4}{x^2+4}$ Label intercepts and point of maximum positive gradient and the straight line $x - 5y = 4$.



3 marks

g. The volume of the solid formed V_x, when the region bound by $y = f(x)$, the two axes and the line $x = a,\ 0 < a \leq 4$ is rotated around the $x -$ axis is $\frac{\pi^2}{16} - \frac{\pi}{8}$. Find the value of a.

2 marks

2 + 2 + 2 + 2 + 2 + 3 + 2 = 15 marks

END OF QUESTION AND ANSWER BOOK