

Question 2 (5 marks)

The quadratic equation: $z^2 + bz + c = 0$, with real coefficients, has a root of $z_1 = 2cis\left(\frac{\pi}{6}\right)$

a. Write down, in polar form, z_2 , the second root to the quadratic equation.

1 mark

b. Find the values of coefficients b and c .

2 marks

TURN OVER

Question 4 (3 marks)

The following approximations may be useful for this question.

Let Z be an observation from a standard normal distribution.

$\Pr(-1 \leq Z \leq 1) \approx 0.68$, $\Pr(-2 \leq Z \leq 2) \approx 0.95$, $\Pr(-3 \leq Z \leq 3) \approx 0.997$

GREENAFIELD produce strips of turf with an advertised length of $1m$. One of the machines cutting the strips is suspected of producing strips that are undersize. A sample of n strips are measured. The results are normally distributed with a mean length of $99cm$ and a standard deviation of $3cm$. It is correctly concluded that there is a 2.5% chance that the actual mean length produced by the machine is above $100cm$. Find n , the number of strips of turf analysed in the sample.

3 marks

Question 5 (4 marks)

Clarry and Tammy are best friends.

Clarry claims that: “The graph of a function can never cross one of its asymptotes.”

Tammy disagrees. To support her opinion, Tammy graphs the function $y = \frac{x^3}{x^2-9}$

- a. Find the equations of all asymptotes for $y = \frac{x^3}{x^2-9}$

2 marks

- b. Find any intercepts for $y = \frac{x^3}{x^2-9}$

1 mark

TURN OVER

c. What conclusion can be drawn regarding Clarry and Tammy’s disagreement?

1 mark

2 + 1 + 1 = 4 marks

Question 7 (8 marks)

Consider the function: $f: [0,1] \rightarrow R$ where $f(x) = \tan^{-1} x$

a. Complete: For a suitable domain, $y = \tan^{-1} x$ and $x = \underline{\hspace{2cm}}$ are equivalent.

1 mark

b. Let $a = f(0)$ and $b = f(1)$. Evaluate a and b .

1 mark

c. By first considering the area bound by $x = \tan y$, the y axis and the line $y = b$, find the area bound by $f(x) = \tan^{-1} x$, the x axis and the line $x = 1$.

3 marks

- d. Find the volume formed when the area bound by $f(x) = \tan^{-1} x$, the y axis and the line $y = b$ is rotated around the y axis.

3 marks

1 + 1 + 3 + 3 = 8 marks

Question 8 (8 marks)

Consider the points in three dimensional space: $A(1,1,-2), B(0,2,-1), C(2,-1,-5)$.

- a. Find \overrightarrow{AB} and \overrightarrow{AC} .

2 marks

TURN OVER

b. Hence find $\vec{AB} \times \vec{AC}$, the cross product of \vec{AB} and \vec{AC} .

2 marks

c. Hence find a unit vector normal to the plane containing points A, B, C .

1 mark

