NAME: ______________________________

VCE® SPECIALIST MATHEMATICS

Units 3 & 4 Practice Written Examination 2

Reading time: 15 minutes
Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	6	6	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 30 pages.
- Formula sheet.
- Answer sheet for multiple-choice questions.

Instructions

- Write your **name** in the space provided above on this page.
- Write your **name** on the multiple-choice answer sheet.
- Unless otherwise indicated, the diagrams are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

 2024-MSP-VIC-U34-NA-EX2-QATS

SECTION A – Multiple-Choice Questions

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for Multiple-Choice Questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1

Consider the following statements:

P: $n^2 - 6n + 5$ is even. Q: n is odd.

Which one of the following statements would be assumed in order to prove the conjecture

If $n^2 - 6n + 5$ is even then *n* is odd

using proof by contradiction?

A. $\neg P \Rightarrow \neg Q$

B. $\neg P \wedge \neg Q$

C. $P \wedge \neg Q$

D. $\neg P \wedge Q$

Question 2

Two of the asymptotes of the graph of $y = \dfrac{|x|^3 - 1}{ax^2 + b|x| + c}$ are given by $y = -2x + 14$ and $x = 6$.

Possible values of *a*, *b* and *c* are, respectively

A. $-\dfrac{1}{2},\ -\dfrac{7}{2},\ -39$

B. $\dfrac{1}{2},\ -\dfrac{7}{2},\ 3$

C. $\dfrac{1}{2},\ \dfrac{7}{2},\ -3$

D. $-2,\ \dfrac{7}{2},\ 3$

Question 3

The graph of $y = a - b^2 \arctan\left(\dfrac{x}{2}\right)$, where $a, b \in R \setminus \{0\}$, will have no x-intercept only when

A. $b^2 \le \left|\dfrac{2a}{\pi}\right|$

B. $b^2 \ge \left|\dfrac{2a}{\pi}\right|$

C. $0 < b^2 \le \dfrac{2a}{\pi}$ and $a > 0$

D. $0 < b^2 < \dfrac{2a}{\pi}$ and $a > 0$

Question 4

The complex numbers z, $-iz$ and $z-iz$, where $z \neq 0$, are plotted in the Argand plane to form the vertices of a triangle.

The area of this triangle is given by

A. $\frac{1}{2}\left(|z|^2+|z|\right)$

B. $\frac{\sqrt{3}}{2}|z|$

C. $\frac{\sqrt{3}}{2}|z|^2$

D. $\frac{1}{2}|z|^2$

Question 5

If $\mathrm{Arg}(z+1)=\frac{\pi}{3}$ and $\mathrm{Arg}(z-1)=\frac{\pi}{2}$, then $\mathrm{Arg}(z-4)$ is equal to

A. $\pi-\arctan\left(\frac{\sqrt{3}}{2}\right)$

B. $-\arctan\left(\frac{\sqrt{3}}{2}\right)$

C. $\pi-\arctan\left(\frac{2\sqrt{3}}{3}\right)$

D. $-\arctan\left(\frac{2\sqrt{3}}{3}\right)$

Question 6

The relation $|z+a|-|z-3a|=k$, where $z \in C$ and $a \in R \setminus \{0\}$, will define a ray when

A. $0 < k \le 3a$

B. $a \le k \le 3a$

C. $k = 3a$

D. $k = 4a$

Question 7

Consider the following pseudocode.

```
define f(t,x)
     return (t^2+1)/(x^3+1)
a ← ___
b ← 4
n ← 10
h ← (b-a)/n
t ← a
x ← 3
for i from 1 to n
     x ← x + h*f(t,x)
     t ← t+h
end for
print x to three decimal places
```

For which one of the following values of a will the pseudocode print `3.504`?

A. 2.1

B. 2.2

C. 2.3

D. 2.4

Question 8

A tank initially contains 16 kg of salt that is dissolved in 90 L of water. A solution containing 6 kg of salt per litre of water flows into the tank at a rate of 5 L per minute and the mixture in the tank is kept well stirred. At the same time, 7 L of the mixture flows out of the tank per minute.
A differential equation defining the concentration, c kg per litre of water, of salt in the tank at time t minutes, for a non-zero volume of mixture, is

A. $\dfrac{dc}{dt} = 30 - 7c$

B. $\dfrac{dc}{dt} = \dfrac{30 - 5c}{90 - 2t}$

C. $\dfrac{dc}{dt} = -\dfrac{7c}{90 - 2t}$

D. $\dfrac{dc}{dt} = \dfrac{30 - 7c}{90 - 2t}$

Question 9

The position vector of a particle moving in the Cartesian plane, at time t, is given by

$$\underset{\sim}{r}(t) = \frac{|3 - t^2|}{t+1}\underset{\sim}{i} - \frac{4}{t+2}\underset{\sim}{j}, \text{ where } t \ge 0.$$

What is the slope of the tangent to the path of the particle when $t = 2$?

A. $\dfrac{9}{44}$

B. $-\dfrac{9}{44}$

C. $\dfrac{11}{36}$

D. $-\dfrac{11}{36}$

Question 10

For n a nonnegative integer, let $I_n = \int_0^{\frac{\pi}{3}} \sec^n(x)\,dx$. If $n \geq 2$, then I_n equals

A. $\dfrac{2^{n-2}}{n-1}\sqrt{3} - \dfrac{n-2}{n-1} I_{n-2}$

B. $\dfrac{2^{n-2}}{n-1}\sqrt{3} + \dfrac{n-2}{n-1} I_{n-2}$

C. $\dfrac{2^{n-2}}{n-1}\sqrt{3} + (n-2) I_{n-2}$

D. $(n-1)2^{n-2}\sqrt{3} I_{n-2}$

Question 11

The area of the surface generated by rotating the curve with equation $y = \tan^{-1}(x)$ about the line $x = 1$ from the points where $x = 2$ to $x = 4$ equals

A. $2\pi \displaystyle\int_{\tan^{-1}(2)}^{\tan^{-1}(4)} \left(\tan(y) - 1\right)\sqrt{1 + \sec^2(y)}\,dy$

B. $2\pi \displaystyle\int_{\frac{\pi}{4}}^{\tan^{-1}(3)} \left(\tan(y) + 1\right)\sqrt{1 + \sec^4(y)}\,dy$

C. $2\pi \displaystyle\int_{\tan^{-1}(2)}^{\tan^{-1}(4)} \left(\tan(y) - 1\right)\sqrt{1 + \sec^4(y)}\,dy$

D. $2\pi \displaystyle\int_{\frac{\pi}{4}}^{\tan^{-1}(3)} \left(\tan(y) + 1\right)\sqrt{1 + \sec^2(y)}\,dy$

Question 12

Consider the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ such that $|\underset{\sim}{a}|=4$, $|\underset{\sim}{b}|=3$ and $|\underset{\sim}{a}-\underset{\sim}{b}|=2$.

Then $|\underset{\sim}{a}+\underset{\sim}{b}|$ equals

A. $\sqrt{21}$

B. 5

C. $\sqrt{38}$

D. $\sqrt{46}$

Question 13

Consider the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ such that $\underset{\sim}{a}\cdot\underset{\sim}{b}=4$ and $|\underset{\sim}{a}\times\underset{\sim}{b}|=7$. The angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$, correct to the nearest degree, is

A. 50

B. 55

C. 60

D. 65

Question 14

A particle starts from rest and moves in a straight line. The velocity, $v\,\mathrm{ms}^{-1}$, of the particle after t seconds is $v=\tan^{-1}(x)$, where x meters is the distance the particle has moved.

If the particle moves 2 metres in 3 seconds, the time it takes to move 5 metres is approximately

A. 1.52 seconds

B. 2.36 seconds

C. 3.52 seconds

D. 5.36 seconds

Question 15

An object with an initial velocity of $6\,\mathrm{ms}^{-1}$ moves in a straight line with constant acceleration. After ten seconds it is moving at $8\,\mathrm{ms}^{-1}$ in the **opposite** direction to its initial velocity.

The distance, in metres, of the object from its starting point after twelve seconds is

A. 28.8

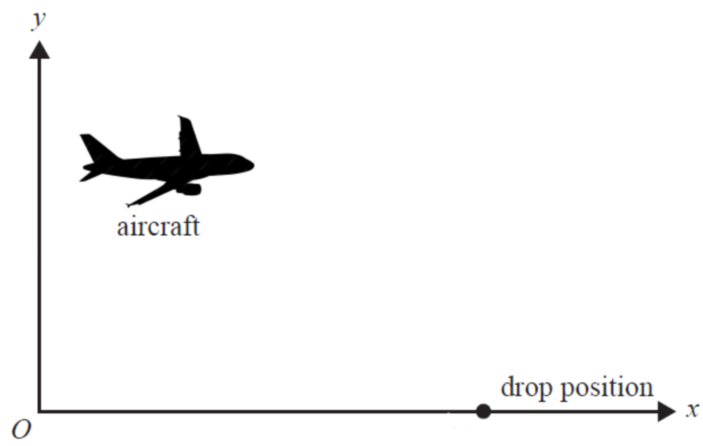
B. 57.6

C. 68.6

D. 86.4

Question 16

A supply aircraft is to release a package so that it lands at a drop position, as shown in the diagram below.



1550 m

The position vector of the supply aircraft relative to a fixed observation point O is

$$\underset{\sim}{r}(t) = 50t\,\underset{\sim}{i} + (600 - 3t)\,\underset{\sim}{j},\quad 0 \le t \le 120 \text{ seconds.}$$

The drop position is located 1550 m horizontally from O. $\underset{\sim}{i}$ is a horizontal unit vector in the direction of the drop position and $\underset{\sim}{j}$ is a unit vector vertically up. Displacement components are measured in metres.

When the aircraft is at a height of 540 m, it releases the package. Assuming negligible air resistance acting on the package, how far short of the drop position, correct to the nearest meter, does the package land?

A. 36

B. 40

C. 44

D. 48

Question 17

The planes

$$2x+(k-4)y+(3-k)z=1, \qquad 2y+(k-3)z=2, \qquad x-2y+z=1$$

intersect along a common line when

A. $k=-1$ and $k=2$

B. $k=2$

C. $k=-1$

D. $k \in R \setminus \{-1,2\}$

Question 18

If the vector $3\underset{\sim}{i}-7\underset{\sim}{j}+\gamma\underset{\sim}{k}$ is perpendicular to the lines $\frac{x-1}{3}=\frac{3-y}{\alpha}=\frac{z+1}{4}$ and $\frac{x+1}{\beta}=\frac{y-2}{3}=\frac{z-1}{5}$, then

A. $\alpha=1$, $\beta=-2$, $\gamma=-4$

B. $\alpha=-1$, $\beta=-2$, $\gamma=-\frac{1}{2}$

C. $\alpha=-2$, $\beta=3$, $\gamma=\frac{5}{4}$

D. $\alpha=-3$, $\beta=2$, $\gamma=3$

Question 19

Consider a random variable X with probability density function

$$f(x)=\begin{cases}2x, & 0\le x\le 1\\ 0, & x<0 \text{ or } x>1\end{cases}.$$

If a large number of samples, each of size 80, is taken from this distribution, then the distribution of sample means, $\overline{X}$, is approximately normal with a standard deviation of

A. $\dfrac{\sqrt{5}}{90}$

B. $\dfrac{\sqrt{5}}{80}$

C. $\dfrac{\sqrt{10}}{120}$

D. $\dfrac{\sqrt{5}}{72}$

Question 20

The lifespan of a certain type of lightbulb is normally distributed with a mean of μ hours and a standard deviation of 420 hours.

A random sample of these lightbulbs is collected and used to calculate an approximate 90% confidence interval for μ. If the confidence interval is (14 428, 14 572) then the sample size is

A. 92

B. 90

C. 88

D. 86

END OF SECTION A

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

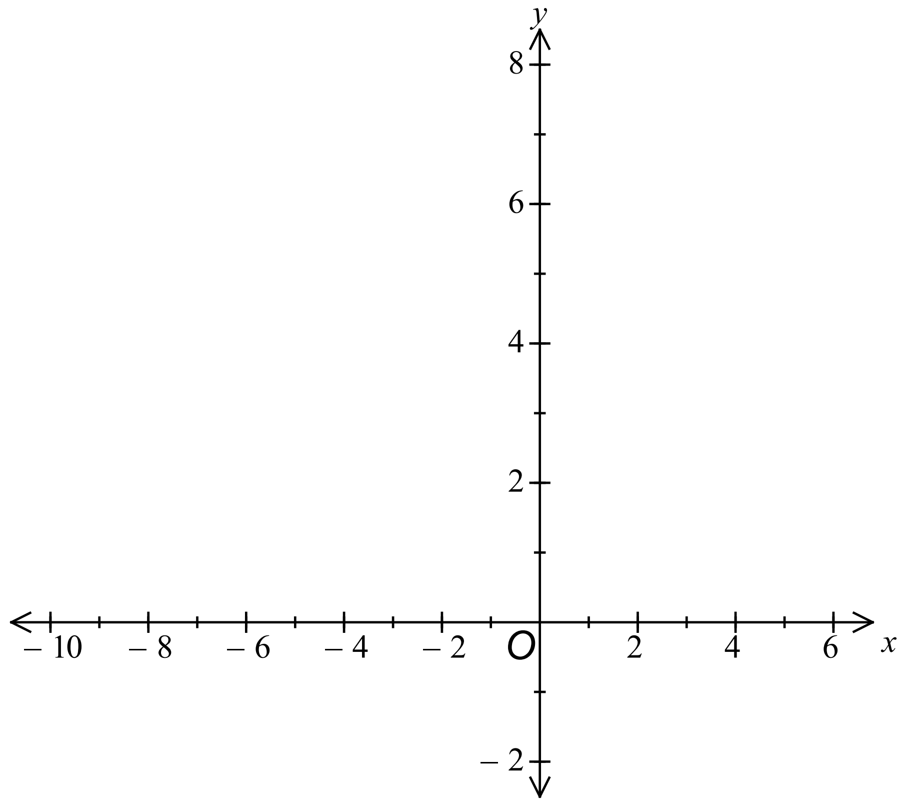
Question 1 (10 marks)

Consider the function f with rule $f(x) = \dfrac{x^2 - a|x| + b}{|x-1|}$ where $a, b \in R$

a. State the equations of all potential asymptotes of f. 2 marks

b. Verify that f is not smooth at the point where $y = b$ except when a has one particular value, which you should state. 3 marks

c. Sketch the graph of $y = f(x)$ on the axes below when $a = 5$ and $b = 6$. Label asymptotes with their equations, and label axis intercepts with their coordinates. 3 marks



d. Consider the function g with rule $g(x) = \frac{x^2 - 5|x| + 6}{|x-1|} - \arcsin(x-1) - k$ for $k \in R$.

Find the values of k for which the graph of $y = g(x)$ has three x-intercepts. Give answers correct to four decimal places if an exact answer is not possible. 2 marks

Question 2 (12 marks)

Let $S = \{z : \ |z-1| = 1, \ z \in C\}$.

a. Express S in cartesian form. 1 mark

b. Find the larger of the two areas bounded by S and $\{z : \ |z| = |z-1|, \ z \in C\}$. 2 marks

c. All except one of the elements of S are elements of

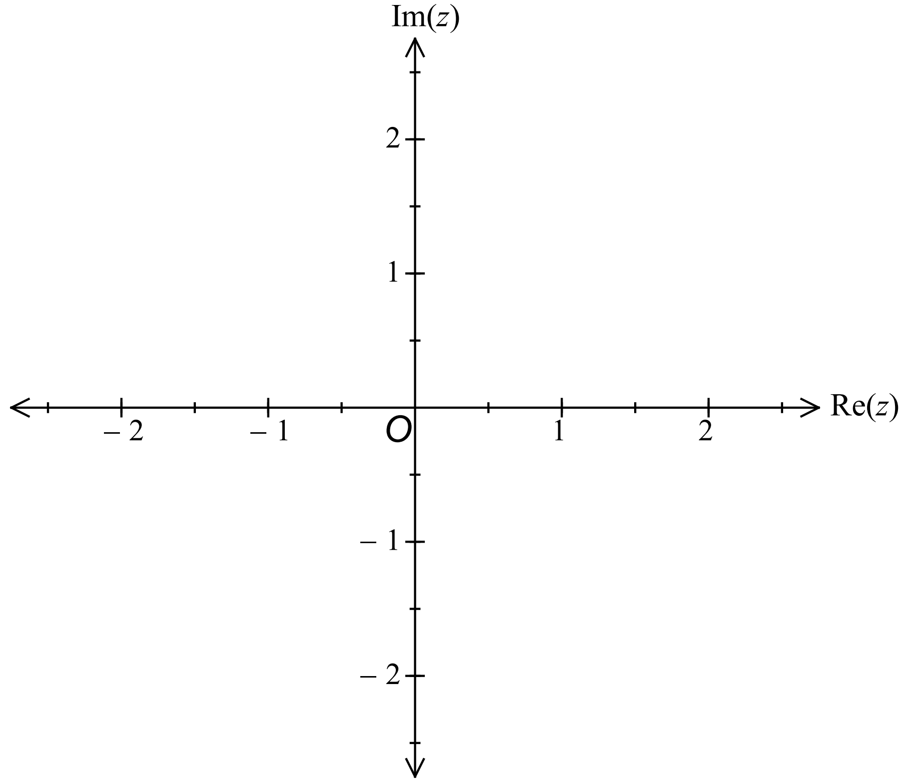
$\left\{z:\ a\mathrm{Arg}\left(1-\frac{b}{z}\right)=\mathrm{Arg}\left(z^2\right),\ z\in C\right\}$ where $a,b\in R^+$

Find the values of a and b. 2 marks

d. Prove that all except one of the elements of S are elements of $\left\{z:\ \mathrm{Arg}(z-1)=\mathrm{Arg}\left(z^{2}\right),\ z\in C\right\}$

2 marks

e. Sketch $\{z: \operatorname{Arg}(z-1)=\operatorname{Arg}(z^2),\ z\in C\}$ on the Argand diagram below. 3 marks



f. Let $z \in S \cap \{z: 3\operatorname{Arg}(z^2)=2\operatorname{Arg}(z^2-z),\ z\in C\}$. Find the values of $\operatorname{Arg}(z)$. 2 marks

Question 3 (9 marks)

Consider the curve given by $y = \sqrt{3 - \tan^{-1}(2x)}$

a. The area bounded by the curve and the x-axis over the interval $x \in [0,\ 1]$ is rotated about the x-axis to form a solid of revolution.

Find, correct to four decimal places, the volume of this solid. 2 marks

b. Find the area of the surface formed when the part of the curve over the interval $x \in [-1,\ 0]$ is rotated about the y-axis. Give your answer correct to four decimal places. 3 marks

c. Find the arclength of the curve over the interval $x \in [-1,\ 1]$. Give your answer correct to four decimal places. 2 marks

Consider the curves $y = f(x)$ and $y = g(x) = f(x) + k$ where $f(x) > 0$ over the interval $x \in [a,\ b]$ and $k \in R^+$

Let L be the arclength of $y = f(x)$ over $x \in [a,\ b]$

Let S_f and S_g be the respective areas of the surface formed when the part of each curve $y = f(x)$ and $y = g(x)$ over the interval $x \in [a,\ b]$ is rotated about the x-axis.

d. Express S_g in terms of S_f and L. 2 marks

Question 4 (10 marks)

At the start of the year 2024 the *Gratuitous Fish Farm* releases 150 fish into a pond that originally contained no fish. Fish are removed from the pond at the end of each year.
The fish population, N, grows according to the model:

$$\frac{dN}{dt} = 2N\left(1 - \frac{N}{600}\right) - n$$

where t is the number of years after the release of the 150 fish and n is the number of fish removed at the end of each year.

a. Find the value(s) of n for which the rate at which the fish population initially grows is positive. 1 mark

b. Find the value(s) of n such that the solution to $\frac{dN}{dt} = 2N\left(1 - \frac{N}{600}\right) - n$ has the form:

$N = 5 + \frac{k}{t - c}$ where $c, k \in R$ 2 marks

c. Find, correct to the nearest integer, the value of n such that there are no fish in the pond after four years. 2 marks

Suppose $n = 100$.

d. Without solving the differential equation:

i. Given that the graph of $N = N(t)$ has a point of inflection, find the N - coordinate of this point. 2 marks

ii. Briefly explain why N has a limiting value and find this value, correct to the nearest integer. 2 marks

e. Find, in years, the value of t when $N = 400$. Give your answer correct to two decimal places. 1 mark

Question 5 (10 marks)

Let the vectors $\underset{\sim}{v} = -\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{w} = 3\underset{\sim}{i} - 2\underset{\sim}{j} - \underset{\sim}{k}$.

a. Show that $\underset{\sim}{v}$ and $\underset{\sim}{w}$ are perpendicular to each other. 1 mark

A plane, Π, contains the point $(2,\ -2,\ -1)$ and the vectors $\underset{\sim}{v}$ and $\underset{\sim}{w}$ are parallel to Π.

b. Find a cartesian equation of Π. 2 marks

The position vector of an object is given by

$$\underset{\sim}{r} = 4\underset{\sim}{i} + 2\underset{\sim}{j} - 3\underset{\sim}{k} + \cos(2t)\underset{\sim}{v} + \sin(3t)\underset{\sim}{w}, \quad 0 \le t \le 2\pi$$

where time t is measured in minutes and distance is measured in decametres.

c. Show that Π contains the path followed by the object. 2 marks

d. Find the distance travelled by the object after one minute. Give your answer in decametres correct to four decimal places. 2 marks

The position vector of a second object is given by

$$\underset{\sim}{s} = \underset{\sim}{i} + \underset{\sim}{j} + 6\underset{\sim}{k} + t\left(-\underset{\sim}{i} + 6\underset{\sim}{j} - \underset{\sim}{k}\right), \quad t \geq 0$$

where time t is measured in minutes and distance is measured in decametres.

e. Find a cartesian equation of the path followed by the second object. 1 mark

f. Find, in decametres, the minimum distance of the second object from Π.
Give your answer in the form $\dfrac{a\sqrt{b}}{3}$ where $a, b \in N$ 2 marks

Question 6 (9 marks)

Gilne Garden Supplies sells large and small bags of manure. It is known that the amount of manure in a small bag is normally distributed with a standard deviation of 0.75 kg.

a. The probability that the combined amount of manure in three small bags is more than 46 kg is approximately 0.6238.

Use an appropriate value of the standard normal random variable Z to find the mean amount of manure in a small bag. Give your answer in kg correct to two decimal places. 3 marks

It is known that the amount of manure in a large bag is normally distributed with a standard deviation of 1 kg. The mean amount of manure in a large bag is advertised to be 25 kg.

Stefan is the manure manager. He thinks that the true mean amount of manure in a large bag is not 25 kg and decides to apply a statistical test at the 3% level of significance.

b. Write down the null hypothesis, H_0, the alternative hypothesis, H_1, and the probability of a type I error for the test. 1 mark

Stefan takes a random sample of 32 large bags and finds that the total amount of manure is 807.4 kg.

c. Find the p value for the test correct to four decimal places. 2 marks

Suppose that the true mean amount of manure in a large bag is 25.6 kg.

d. Calculate the probability, correct to three decimal places, of Stefan making a type II error in the statistical test. 3 marks

END OF EXAMINATION

Multiple-Choice Answer Sheet

Student Name:

Shade the letter that corresponds to each correct answer.

Question	A	B	C	D
Question 1	[ ]	[ ]	[ ]	[ ]
Question 2	[ ]	[ ]	[ ]	[ ]
Question 3	[ ]	[ ]	[ ]	[ ]
Question 4	[ ]	[ ]	[ ]	[ ]
Question 5	[ ]	[ ]	[ ]	[ ]
Question 6	[ ]	[ ]	[ ]	[ ]
Question 7	[ ]	[ ]	[ ]	[ ]
Question 8	[ ]	[ ]	[ ]	[ ]
Question 9	[ ]	[ ]	[ ]	[ ]
Question 10	[ ]	[ ]	[ ]	[ ]
Question 11	[ ]	[ ]	[ ]	[ ]
Question 12	[ ]	[ ]	[ ]	[ ]
Question 13	[ ]	[ ]	[ ]	[ ]
Question 14	[ ]	[ ]	[ ]	[ ]
Question 15	[ ]	[ ]	[ ]	[ ]
Question 16	[ ]	[ ]	[ ]	[ ]
Question 17	[ ]	[ ]	[ ]	[ ]
Question 18	[ ]	[ ]	[ ]	[ ]
Question 19	[ ]	[ ]	[ ]	[ ]
Question 20	[ ]	[ ]	[ ]	[ ]