NAME: _____

VCE®SPECIALIST MATHEMATICS

Units 3 & 4 Practice Written Examination 1

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are **NOT** permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and Answer Booklet of 14 pages.
- Formula Sheet.
- Working space is provided throughout the Question and Answer Booklet.

Instructions
- Write your **student name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the space provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

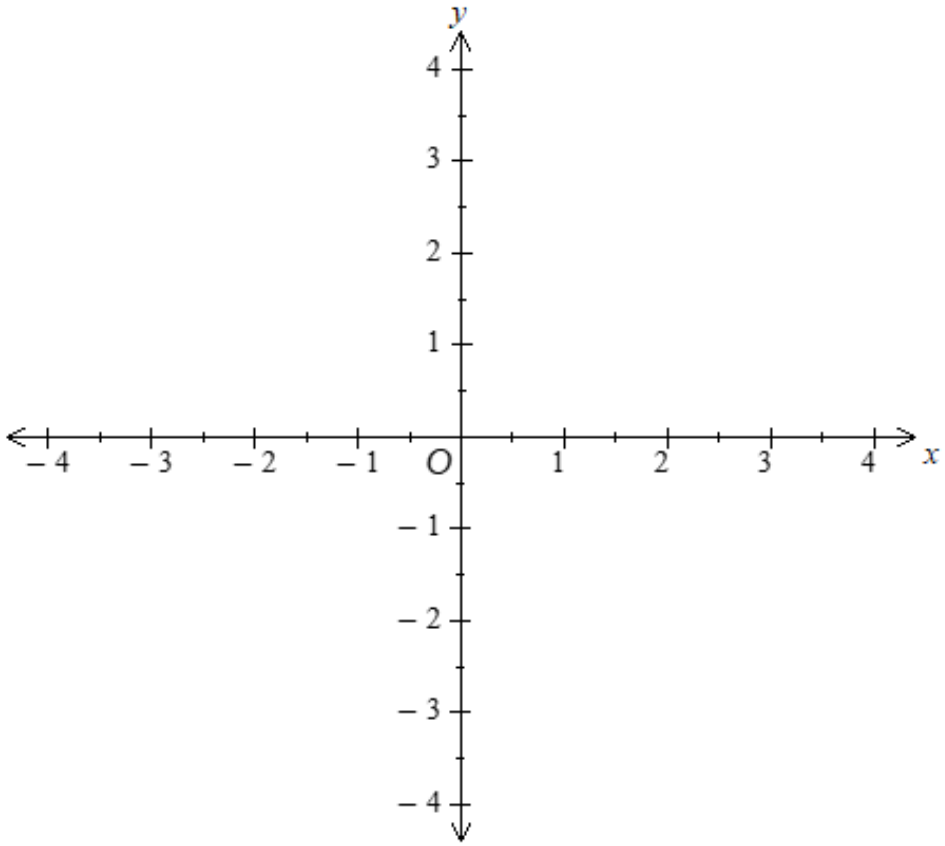
Take the **acceleration due to gravity** to have magnitude g m s^{-2} where $g = 9.8$.

Question 1 (5 marks)

Consider the function f_k with rule $f_k(x) = \dfrac{x^3 - x^2 + kx + 1}{x^2 + 1}$ where k is a real constant.

a. Find the value(s) of k for which the graph of f_k does not intersect its asymptote. 2 marks

b. Sketch the graph of f_k for $k = 3$ on the axes below. Label any asymptotes with their equation and any turning points and intersection with asymptotes with their coordinates. 3 marks



Working space

Question 2 (4 marks)

Solve $3z^2 - 2\bar{z} = |2z - 1|$ where $z \in C$

Question 3 (4 marks)

A particle with an initial velocity of 1 ms^{-1} moves in a straight line. When the particle is x m from its starting point its velocity, v ms^{-1}, after t seconds is given by

$$v = 2 - e^{-x}, \text{ where } x \geq 0.$$

Find the position of the particle when $t = \log_e(7)$.

Question 4 (3 marks)

A particle moves in a straight line such that after t seconds its displacement x meters from a fixed origin O is given by $t^3 + 2x^3 - xt = 2$.

Find the acceleration of the particle when $t = 1$ and $x = 1$.

Question 5 (3 marks)

Evaluate $\int_0^{2\sqrt{3}} x \tan^{-1}\left(\frac{x}{2}\right) dx$

Question 6 (4 marks)

The *Gratuitous Apple Farm* grows apples. It is known that the standard deviation of the mass of the apples is 5 grams. A random sample of 64 apples is used to calculate an approximate $C\%$ confidence interval for the mean mass, measured in grams, of all apples grown on the farm.

The confidence interval was found to be (201.2, 202.6).

a. $C = 100\Pr\left(|Z| < k\right)$ where Z is the standard normal random variable.

Find the value of k. 1 mark

A larger sample of apples is selected, with a sample size four times the original sample. The sample mean is found to be the same.

b. State the approximate $C\%$ confidence interval for the mean mass, measured in grams, of all apples grown on the farm that would be calculated using this sample. Give all values correct to one decimal place. 1 mark

The *Gratuitous Apple Farm* also grows a small crop of genetically modified apples. The mass of these apples is normally distributed with a mean of 144 grams and a standard deviation of 8 grams.

c. Two genetically modified apples are chosen at random and compared to a third genetically modified apple. Find the probability that the combined mass of the two chosen apples is less than $\frac{7}{4}$ of the mass of the third apple. Use $\Pr(|Z| \leq 2) = 0.9545$ and give your answer correct to three decimal places. 2 marks

Question 7 (4 marks)

The curve defined by the parametric equations

$$x = \cos^3(t), \quad y = \sin^3(t), \text{ where } 0 \le t \le \frac{\pi}{2}$$

is rotated about the *y*-axis.

Find the area of the surface generated.

Question 8 (3 marks)

Use mathematical induction to prove that for $n \geq 2$,

$$\frac{\left(f_1(x) \times \cdots \times f_n(x)\right)'}{f_1(x) \times \cdots \times f_n(x)} = \frac{f_1'(x)}{f_1(x)} + \cdots + \frac{f_n'(x)}{f_n(x)}$$

for all sets of differentiable functions $f_1(x),\ f_2(x), \ldots,\ f_n(x)$.

Question 9 (5 marks)

Consider the following planes:

$$\Pi_1 : \ 3x - 2y - mz = 2$$

$$\Pi_2 : \ x + my + 3mz = 4$$

a. Find the value(s) of m that will make Π_1 and Π_2 perpendicular to each other. 2 marks

b. Let $m = -1$.

Find a vector perpendicular to Π_1 and the line $x + 1 = \frac{2 - y}{2} = \frac{z - 3}{2}$ 3 marks

Question 10 (5 marks)

The velocity vector of a particle at time t seconds is given by

$$\underset{\sim}{v}(t) = \sin^2(t)\,\underset{\sim}{i} + \sin(t)\,\underset{\sim}{k}$$

The particle is initially at the point (0, 1, 0).

a. Find the position vector of the particle after t seconds. 2 marks

The position vector of a second particle at time t seconds is given by

$$\underset{\sim}{s}(t) = \cos(2t)\underset{\sim}{i} - 3\underset{\sim}{j} + \left(2t - \sin(2t)\right)\underset{\sim}{k}$$

b. Find the times at which the two particles are moving in the same direction. 3 marks

END OF EXAMINATION