VCE® Specialist Mathematics

Practice Written Examination 1

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Solution Pathway

Below are sample answers. Please consider the merit of alternative responses.

Specialist Mathematics Examination 1: Marking Scheme

Question 1

Part a.:

A function can intersect with either a horizontal asymptote or a diagonal (oblique, slant) asymptote:

$$\begin{array}{r} x-1 \\ x^2+1 \overline{\big) x^3 - x^2 + kx + 1} \\ -\left(x^3 + 0x^2 + x\right) \\ \hline -x^2 + (k-1)x + 1 \\ -\left(-x^2 + 0x - 1\right) \\ \hline (k-1)x + 2 \end{array}$$

Therefore $f_k(x) = \dfrac{x^3 - x^2 + kx + 1}{x^2+1} = x - 1 + \dfrac{(k-1)x+2}{x^2+1}$

Diagonal asymptote: $y = x - 1$	**1 mark**

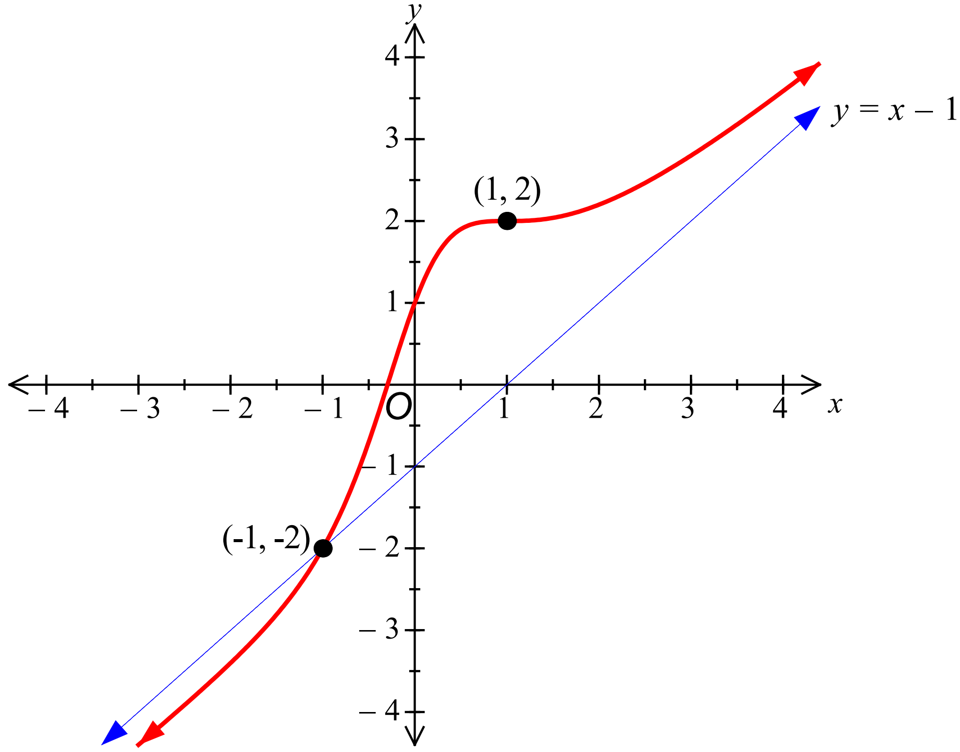
Solve $f_k(x) = x - 1 + \dfrac{(k-1)x+2}{x^2+1} = x - 1$: $\dfrac{(k-1)x+2}{x^2+1} = 0 \quad \Rightarrow (k-1)x + 2 = 0$.

By inspection there is no solution when $k = 1$

Answer: $k = 1$	**1 mark**

Part b.:

Answer:



Shape: Must have a y-intercept at $(0,\ 1)$	**1 mark**
Turning point: $(1,\ 2)$	**1 mark**
Diagonal asymptote: $y = x - 1$ Graph must be consistent with axes scale. **Intersection point:** $(-1,\ -2)$	**1 mark**

Calculations:

- Diagonal asymptote: $y = x - 1$ (from **part a.**).

- Intersection with asymptote:

Substitute $k = 3$ into $\frac{(k-1)x+2}{x^2+1} = 0$ from **part a.**: $2x + 2 = 0 \Rightarrow x = -1$. $y = f_3(-1) = -2$

- Approach towards diagonal asymptote:

$y = x - 1 + \frac{2x+2}{x^2+1}$ (from **part a**.) $\sim x + 0^+$ as $x \to +\infty$

therefore, the approach is from above.

$y = x - 1 + \frac{2x+2}{x^2+1}$ (from **part a**.) $\sim x + 0^-$ as $x \to -\infty$

therefore, the approach is from below.

The approach can also be deduced from the fact that the graph must intersect the diagonal asymptote.

- Turning point: Solve $f_3'(x) = 0$.

$$f_3'(x) = \frac{\left(3x^2 - 2x + 3\right)\left(x^2+1\right) - 2x\left(x^3 - x^2 + 3x + 1\right)}{\left(x^2+1\right)^2} \qquad = \frac{x^4 - 4x + 3}{\left(x^2+1\right)^2}$$

$\frac{x^4 - 4x + 3}{\left(x^2+1\right)^2} = 0 \qquad \Rightarrow x^4 - 4x + 3 = 0 \qquad \Rightarrow x = 1$ (by inspection). $y = f_3(1) = 2$

Question 2

Substitute $z = x + iy$, $x, y \in R$:

$$3\left(x^2 + 2xyi - y^2\right) - 2(x - iy) = |2(x + iy) - 1|$$

$$\Rightarrow 3x^2 + 6xyi - 3y^2 - 2x + 2yi) = |(2x - 1) + 2yi|$$

$$\Rightarrow \left(3x^2 - 3y^2 - 2x\right) + i(6xy + 2y) = \underbrace{|(2x - 1) + 2yi|}_{\text{Real}} \qquad (1)$$

Equate real parts of (1): $3x^2 - 3y^2 - 2x = |(2x - 1) + 2yi|$ (2)

Equate imaginary parts of (1):

$6xy + 2y = 0 \Rightarrow 2y(3x + 1) = 0$ therefore $y = 0$ or $x = -\frac{1}{3}$

Case 1: $y = 0$. Substitute $y = 0$ into equation (2): $3x^2 - 2x = \|2x - 1\|$	**1 mark**

Case 1 (a): $2x - 1 \ge 0 \Rightarrow x \ge \frac{1}{2}$

$3x^2 - 2x = 2x - 1 \quad \Rightarrow 3x^2 - 4x + 1 = 0 \quad \Rightarrow (3x - 1)(x - 1) = 0 \quad \Rightarrow x = 1$

($x = \frac{1}{3}$ is rejected because $x \ge \frac{1}{2}$)

Answer: $z = 1$	**1 mark**

Case 1 (b): $2x - 1 < 0 \Rightarrow x < \frac{1}{2}$

$3x^2 - 2x = -(2x - 1) \quad \Rightarrow 3x^2 = 1 \quad \Rightarrow x = -\frac{1}{\sqrt{3}}$

($x = \frac{1}{\sqrt{3}}$ is rejected because $x < \frac{1}{2}$).

Answer: $z = -\frac{1}{\sqrt{3}}$.	**1 mark**

Case 2: $x = -\frac{1}{3}$ Substitute into equation (2) and simplify: $1-3y^2 = \left|-\frac{5}{3}+2yi\right| \quad \Rightarrow 1-3y^2 = \sqrt{\frac{25}{9}+4y^2}$ There is no real solution (proof is not required).	**1 mark**

The fact that there is no real solution can be seen by inspecting the graphs of $f(y) = 1-3y^2$ and $g(y) = \sqrt{\frac{25}{9}+4y^2}$ and noting that the graphs do not intersect each other.

Question 3

$v = \frac{dx}{dt} = 2 - e^{-x} \qquad \Rightarrow \frac{dt}{dx} = \frac{1}{2 - e^{-x}}$

$\Rightarrow t = \int \frac{1}{2 - e^{-x}}\, dx$	**1 mark**

$= \int \frac{e^x}{2e^x - 1}\, dx$.

Substitute $u = e^x$:

$t = \int \frac{1}{2u - 1}\, du \qquad = \frac{1}{2}\log_e |2u - 1| + c$, where $c \in R$

$t = \frac{1}{2}\log_e |2e^x - 1| + c$	**1 mark**

$\Rightarrow e^{2(t-c)} = |2e^x - 1| \qquad \Rightarrow e^{-2c}e^{2t} = |2e^x - 1| \qquad \Rightarrow \pm e^{-2c}e^{2t} = 2e^x - 1$

$\Rightarrow Ae^{2t} = 2e^x - 1$, where $A = \pm e^{-2c}$.

Substitute $x = 0$ when $t = 0$: $A = 1$.

$e^{2t} = 2e^x - 1$	**1 mark**

Substitute $t = \log_e(7)$:

$2e^x - 1 = e^{2\log_e(7)} = e^{\log_e(49)} = 49$

$\Rightarrow 2e^x = 50 \qquad \Rightarrow e^x = 25$

Answer: $x = \log_e(25)$ Accept $x = 2\log_e(5)$	**1 mark**

Question 4

The value of $a = \dfrac{d^2x}{dt^2}$ when $t = 1$ and $x = 1$ is required.

Use implicit differentiation:

Differentiate $t^3 + 2x^3 - xt = 2$ with respect to *t*: $3t^2 + \underbrace{6x^2\dfrac{dx}{dt}}_{\text{Chain Rule}} \underbrace{-x - t\dfrac{dx}{dt}}_{\text{Product Rule}} = 0$

Result of implicit differentiation: $3t^2 + 6x^2\dfrac{dx}{dt} - x - t\dfrac{dx}{dt} = 0$	**1 mark**

Substitute $t = 1$ and $x = 1$: $3 + 6\dfrac{dx}{dt} - 1 - \dfrac{dx}{dt} = 0 \quad \Rightarrow \dfrac{dx}{dt} = -\dfrac{2}{5}$.

Differentiate $3t^2 + 6x^2\dfrac{dx}{dt} - x - t\dfrac{dx}{dt} = 0$ with respect to *t*:

$$6t + \underbrace{\left(\underbrace{12x\frac{dx}{dt}}_{\text{Chain Rule}}\right)\frac{dx}{dt} + 6x^2\frac{d^2x}{dt^2}}_{\text{Product Rule}} - \frac{dx}{dt} \underbrace{- \frac{dx}{dt} - t\frac{d^2x}{dt^2}}_{\text{product Rule}} = 0$$

Result of implicit differentiation: $6t + 12x\dfrac{dx}{dt} \times \dfrac{dx}{dt} + 6x^2\dfrac{d^2x}{dt^2} - \dfrac{dx}{dt} - \dfrac{dx}{dt} - t\dfrac{d^2x}{dt^2} = 0$.	**1 mark**

Substitute $t = 1$ and $x = 1$ and $\dfrac{dx}{dt} = -\dfrac{2}{5}$:

$$6 + 12\left(-\frac{2}{5}\right)\left(-\frac{2}{5}\right) + 6\frac{d^2x}{dt^2} - \left(-\frac{2}{5}\right) - \left(-\frac{2}{5}\right) - \frac{d^2x}{dt^2} = 0 \qquad \Rightarrow 6 + \frac{48}{25} + 5\frac{d^2x}{dt^2} + \frac{4}{5} = 0$$

Answer: $a = -\dfrac{218}{125}$ m s^{-2}	**1 mark**

Question 5

Use integration by parts to calculate $\int x\tan^{-1}\left(\frac{x}{2}\right)dx$:

$$\int u\,dv = uv - \int v\,du$$

$$u = \tan^{-1}\left(\frac{x}{2}\right) \qquad \Rightarrow du = \frac{2}{x^2+4}dx$$

$$dv = x\,dx \qquad \Rightarrow v = \frac{1}{2}x^2$$

$\int x\tan^{-1}\left(\frac{x}{2}\right)dx = \frac{1}{2}x^2\tan^{-1}\left(\frac{x}{2}\right) - \int \frac{x^2}{x^2+4}dx$	**1 mark**

$$= \frac{1}{2}x^2\tan^{-1}\left(\frac{x}{2}\right) - \int \frac{\left(x^2+4\right)-4}{x^2+4}dx \qquad = \frac{1}{2}x^2\tan^{-1}\left(\frac{x}{2}\right) - \int 1 - \frac{4}{x^2+4}dx$$

$= \frac{1}{2}x^2\tan^{-1}\left(\frac{x}{2}\right) - x + 2\tan^{-1}\left(\frac{x}{2}\right)$	**1 mark**

Therefore:

$$\int_0^{2\sqrt{3}} x\tan^{-1}\left(\frac{x}{2}\right)dx = \frac{1}{2}(12)\tan^{-1}\left(\sqrt{3}\right) - 2\sqrt{3} + 2\tan^{-1}\left(\sqrt{3}\right) - 0 \qquad = 6\left(\frac{\pi}{3}\right) - 2\sqrt{3} + 2\left(\frac{\pi}{3}\right)$$

Answer: $\frac{8\pi}{3} - 2\sqrt{3}$ Accept $\frac{8\pi - 6\sqrt{3}}{3}$	**1 mark**

Question 6

Confidence interval endpoints of a C% confidence interval:

$$\bar{x} \pm z_c \frac{\sigma}{\sqrt{n}}$$

where z_c is such that $\Pr\left(-z_c < Z < z_c\right) = \Pr\left(|Z| < z_c\right) = \frac{C}{100}$

Part a.:

Compare the endpoints of the given confidence interval (201.2, 202.6) with $\bar{x} \pm z_c \frac{\sigma}{\sqrt{n}}$

It follows that:

$$202.6 - 201.2 = 2z_c \frac{\sigma}{\sqrt{n}} \qquad \Rightarrow 1.4 = 2k\frac{5}{\sqrt{64}} = \frac{5}{4}k \qquad \Rightarrow k = \frac{4(1.4)}{5} = \frac{28}{25}$$

Answer: $k = \frac{28}{25}$ Accept 1.12	**1 mark**

Part b.:

Method 1:

Compare the endpoints of the given confidence interval (201.2, 202.6) with $\bar{x} \pm z_c \frac{\sigma}{\sqrt{n}}$

It follows that:

$$202.6 + 201.2 = 2\bar{x} \quad \Rightarrow 403.8 = 2\bar{x} \quad \Rightarrow \bar{x} = 201.9$$

Substitute $\bar{x} = 201.9$, $n = 4 \times 64$, $\sigma = 5$ and $z_c = k = 1.12$ into $\left(\bar{x} - z_c \frac{\sigma}{\sqrt{n}},\ \bar{x} + z_c \frac{\sigma}{\sqrt{n}}\right)$

Method 2:

Let $n' = 4n$. Width of new confidence interval:

$$2k\frac{\sigma}{\sqrt{n'}} = 2k\frac{\sigma}{\sqrt{4n}} = \frac{1}{2}\left(\underbrace{2k\frac{\sigma}{\sqrt{n}}}_{\substack{\text{Width of old}\\ \text{confidence interval}}}\right) = \frac{1}{2}(1.4) = 0.7$$

$\bar{x}$ remains the same therefore the new confidence interval is

$$\left(201.9 - \frac{1}{2}(0.7),\ 201.9 + \frac{1}{2}(0.7)\right)$$

Answer: (201.55, 202.25)	**1 mark**

Part c.:

Let X be the random variable "*Mass of a genetically modified apple*".

X is normally distributed with a mean of 144 grams and a standard deviation of 8 grams.

$\Pr\left(X_1 + X_2 < \frac{7}{4}X_3\right) = \Pr\left(X_1 + X_2 - \frac{7}{4}X_3 < 0\right)$ is required

where X_1, X_2, X_3 are independent copies of X.

Let Y be the random variable $Y = X_1 + X_2 - \frac{7}{4}X_3$. Then $\Pr(Y < 0)$ is required.

Note: Using the random variable $X + X - \frac{7}{4}X = \frac{1}{4}X$ is incorrect: $X_1 + X_2 - \frac{7}{4}X_3 \neq \frac{1}{4}X$

Y follows a normal distribution since X_1, X_2, X_3 are independent normal random variables.

$$\text{E}(Y) = \mu_Y = \mu_{X_1} + \mu_{X_2} - \frac{7}{4}\mu_{X_3} = \frac{1}{4}\mu_X = \frac{1}{4}(144) = 36$$

$$\text{Var}(Y) = 1^2\text{Var}(X_1) + 1^2\text{Var}(X_2) + \left(\frac{7}{4}\right)^2\text{Var}(X_3) = 2\text{Var}(X) + \frac{49}{16}\text{Var}(X) = \frac{81}{16}\text{Var}(X)$$

$$\Rightarrow \text{sd}(Y) = \sigma_Y = \frac{9}{4}\text{sd}(X) = \frac{9}{4}(8) = 18$$

$Y = X_1 + X_2 - \frac{7}{4}X_3$ $Y \sim \text{Normal}\left(\mu_Y = 36,\ \sigma_Y = 18\right)$ $\Pr(Y < 0)$	**1 mark:** Definition. Distribution. Probability.

$Z = \frac{Y - \mu_Y}{\sigma_Y} = \frac{0 - 36}{18} = -2$ therefore $\Pr(Y < 0) = \Pr(Z < -2)$.

$\Pr(|Z| \le 2) = 0.9545 \Rightarrow \Pr(Z < -2) = \frac{1}{2}(1 - 0.9545) = \frac{1}{2}(0.0455)$.

$Z = -2$. **Answer:** 0.023	**1 mark:** Calculation of Z. Answer.

Question 7

$$S = 2\pi \int_0^{\frac{\pi}{2}} x\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

Substitute $\frac{dx}{dt} = -3\cos^2(t)\sin(t)$ and $\frac{dy}{dt} = 3\sin^2(t)\cos(t)$:

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 9\cos^4(t)\sin^2(t) + 9\sin^4(t)\cos^2(t) \qquad = 9\cos^2(t)\sin^2(t)\left(\cos^2(t) + \sin^2(t)\right)$$

$$= 9\cos^2(t)\sin^2(t)$$

$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 9\cos^2(t)\sin^2(t)$	**1 mark**

$S = 2\pi \int_0^{\frac{\pi}{2}} \cos^3(t)\sqrt{9\cos^2(t)\sin^2(t)}\,dt$	**1 mark**

$$= 6\pi \int_0^{\frac{\pi}{2}} \cos^3(t)\,|\cos(t)\sin(t)|\,dt \qquad = 6\pi \int_0^{\frac{\pi}{2}} \cos^3(t)\cos(t)\sin(t)\,dt$$

Since $\cos(t)\sin(t) \ge 0$ for $0 \le t \le \frac{\pi}{2}$ $= 6\pi \int_0^{\frac{\pi}{2}} \cos^4(t)\sin(t)\ dt$	**1 mark:** Justification for dropping modulus. Integral.

Substitute $u = \cos(t)$:

$$S = -6\pi \int_1^0 u^4\ du \qquad = 6\pi \int_0^1 u^4\ du \qquad = \frac{6\pi}{5}\left[u^5\right]_0^1.$$

Answer: $\frac{6\pi}{5}$ square units.	**1 mark**

Question 8

• Let $S(n)$ be the conjecture $\dfrac{\left(f_1(x)\ldots f_n(x)\right)'}{f_1(x)\ldots f_n(x)} = \dfrac{f_1'(x)}{f_1(x)} + \ldots + \dfrac{f_n'(x)}{f_n(x)}$ • Check $n=2$: LHS $= \dfrac{\left(f_1(x)f_2(x)\right)'}{f_1(x)f_2(x)} = \dfrac{f_1'(x)f_2(x)+f_1(x)f_2'(x)}{f_1(x)f_2(x)} = \dfrac{f_1'(x)}{f_1(x)} + \dfrac{f_2'(x)}{f_2(x)} =$ RHS Therefore $S(2)$ is true. • Assume $S(k)$ is true for some $k>2 \in N$: $\dfrac{\left(f_1(x)\ldots f_k(x)\right)'}{f_1(x)\ldots f_k(x)} = \dfrac{f_1'(x)}{f_1(x)} + \ldots + \dfrac{f_k'(x)}{f_k(x)}$	**1 mark:** Statement. Base case. Inductive hypothesis.

• Show that if $S(k)$ is true then it follows that $S(k+1)$ is true: $\dfrac{\left(f_1(x)\ldots f_k(x)f_{k+1}(x)\right)'}{f_1(x)\ldots f_k(x)f_{k+1}(x)} = \dfrac{\left(\left[f_1(x)\ldots f_k(x)\right]f_{k+1}(x)\right)'}{f_1(x)\ldots f_k(x)f_{k+1}(x)}$ $= \dfrac{\left(f_1(x)\ldots f_k(x)\right)' f_{k+1}(x) + \left(f_1(x)\ldots f_k(x)\right) f_{k+1}'(x)}{f_1(x)\ldots f_k(x)f_{k+1}(x)}$	**1 mark:** Statement. Recognition that $f_1(x)\ldots f_k(x)f_{k+1}(x)$ is a product of the two functions $f_1(x)\ldots f_k(x)$ and $f_{k+1}(x)$.

$$=\frac{\left(\frac{f_1'(x)}{f_1(x)}+\cdots+\frac{f_k'(x)}{f_k(x)}\right)\left(f_1(x)\ldots f_k(x)\right)f_{k+1}(x)+\left(f_1(x)\ldots f_k(x)\right)f_{k+1}'(x)}{f_1(x)\ldots f_k(x)f_{k+1}(x)}$$

Using the inductive hypothesis

$$=\frac{\left(\frac{f_1'(x)}{f_1(x)}+\cdots+\frac{f_k'(x)}{f_k(x)}\right)f_1(x)\ldots f_k(x)f_{k+1}(x)}{f_1(x)\ldots f_k(x)f_{k+1}(x)}+\frac{f_1(x)\ldots f_k(x)f_{k+1}'(x)}{f_1(x)\ldots f_k(x)f_{k+1}(x)}$$

$$=\left(\frac{f_1'(x)}{f_1(x)}+\cdots+\frac{f_k'(x)}{f_k(x)}\right)+\frac{f_{k+1}'(x)}{f_{k+1}(x)}$$

Which is equivalent to $S(k+1)$ true.

- Since $S(2)$ is true and it follows that if $S(k)$ is true then $S(k+1)$ is true, it follows from the principle of mathematical induction that $S(n)$ is true for $n \geq 2$.

1 mark:

Use of inductive hypothesis.

$S(k) \Rightarrow S(k+1)$.

Conclusion.

Question 9

Part a.:

$\Pi_1: \ \underset{\sim}{n}_1 = 3\underset{\sim}{i} - 2\underset{\sim}{j} - m\underset{\sim}{k}$

$\Pi_2: \ \underset{\sim}{n}_2 = \underset{\sim}{i} + m\underset{\sim}{j} + 3m\underset{\sim}{k}$

Π_1 and Π_2 are perpendicular when $\underset{\sim}{n}_1 \cdot \underset{\sim}{n}_2 = 0$:

$3 - 2m - 3m^2 = 0$	**1 mark**

$\Rightarrow 3m^2 + 2m - 3 = 0 \qquad \Rightarrow m = \frac{-2 \pm \sqrt{4+36}}{6} = \frac{-2 \pm 2\sqrt{10}}{6} = \frac{-1 \pm \sqrt{10}}{3}$

Answer: $m = \frac{-1 \pm \sqrt{10}}{3}$	**1 mark**

Part b.:

$\Pi_1: \ \underset{\sim}{n}_1 = 3\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$

Parametric equations of the line $x+1 = \dfrac{2-y}{2} = \dfrac{z-3}{2}$:

$x+1 = t \qquad \Rightarrow x = -1+t$

$\dfrac{2-y}{2} = t \qquad \Rightarrow y = 2-2t$

$\dfrac{z-3}{2} = t \qquad \Rightarrow z = 3+2t$

Vector in direction of the line (by inspection of the coefficients of *t* in the parametric equations):

$\underset{\sim}{l} = \underset{\sim}{i} - 2\underset{\sim}{j} + 2\underset{\sim}{k}$	**1 mark:**

Vector perpendicular to $\underset{\sim}{n}_1 = 3\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{l} = \underset{\sim}{i} - 2\underset{\sim}{j} + 2\underset{\sim}{k}$:

$$\underset{\sim}{n}_1 \times \underset{\sim}{l} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 3 & -2 & 1 \\ 1 & -2 & 2 \end{vmatrix}$$

$= \underset{\sim}{i}\begin{vmatrix} -2 & 1 \\ -2 & 2 \end{vmatrix} - \underset{\sim}{j}\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} + \underset{\sim}{k}\begin{vmatrix} 3 & -2 \\ 1 & -2 \end{vmatrix}$	**1 mark**

Answer: $-2\underset{\sim}{i} - 5\underset{\sim}{j} - 4\underset{\sim}{k}$.	**1 mark**

Question 10

Part a.:

$\underset{\sim}{r}(t) = \int \sin^2(t)\ dt\ \underset{\sim}{i} + \int \sin(t)\ dt\ \underset{\sim}{k}$ $\quad$ $\underset{\sim}{r}(t) = \frac{1}{2}\int 1 - \cos(2t)\ dt\ \underset{\sim}{i} + \int \sin(t)\ dt\ \underset{\sim}{k}$

$= \left(\frac{t}{2} - \frac{1}{4}\sin(2t)\right)\underset{\sim}{i} - \cos(t)\underset{\sim}{k} + \underset{\sim}{c}$	**1 mark**

where $\underset{\sim}{c}$ is an arbitrary constant vector.

The particle is initially at the point (0, 1, 0) therefore $\underset{\sim}{r}(0) = 0\underset{\sim}{i} + \underset{\sim}{j} + 0\underset{\sim}{k}$.

But $\underset{\sim}{r}(0) = \left(0 - \frac{1}{4}\sin(0)\right)\underset{\sim}{i} - \cos(0)\underset{\sim}{k} + \underset{\sim}{c} = -\underset{\sim}{k} + \underset{\sim}{c}$

Therefore: $0\underset{\sim}{i} + \underset{\sim}{j} + 0\underset{\sim}{k} = -\underset{\sim}{k} + \underset{\sim}{c}$ $\quad \Rightarrow \underset{\sim}{c} = \underset{\sim}{j} + \underset{\sim}{k}$

Answer: $\underset{\sim}{r}(t) = \left(\frac{t}{2} - \frac{1}{4}\sin(2t)\right)\underset{\sim}{i} + \underset{\sim}{j} + \left(1 - \cos(t)\right)\underset{\sim}{k}$	**1 mark**

Part b.:

The particles are moving in the same direction when their velocity vectors are equal:

Particle 1: $\underset{\sim}{v}(t) = \sin^2(t)\underset{\sim}{i} + \sin(t)\underset{\sim}{k}$

Particle 2: $\frac{d\underset{\sim}{s}}{dt} = \underset{\sim}{v}_2(t) = -2\sin(2t)\underset{\sim}{i} + \left(2 - 2\cos(2t)\right)\underset{\sim}{k}$

Equate velocity components:

$\sin^2(t) = -2\sin(2t)$ (1) $\sin(t) = 2 - 2\cos(2t)$ (2)	**1 mark:** Both equations.

Simultaneous solutions to equations (1) and (2) are required. From equation (1):

$\sin^2(t) + 2\sin(2t) = 0 \quad \Rightarrow \sin^2(t) + 4\sin(t)\cos(t) = 0 \quad \Rightarrow \sin(t)(\sin(t) + 4\cos(t)) = 0$

Case 1 $\sin(t) = 0 \quad \Rightarrow t = n\pi,\ n \in Z^+ \cup \{0\}$. **Note:** $t \geq 0$ **Case 2:** $\sin(t) + 4\cos(t) = 0 \quad \Rightarrow \tan(t) = -4$	**1 mark:** Both cases.

Substitute **case 1** solutions into (2): $\sin(n\pi) = 2 - 2\cos(2n\pi) \quad \Rightarrow 0 = 0$ true.

Therefore accept **case 1** solutions.

Substitute **case 2** solutions into (2):

$\tan(t) = -4$ therefore $\sin(t) = \pm\frac{4}{\sqrt{17}}$ and $\cos(2t) = 1 - 2\sin^2(t) = 1 - \frac{8}{\sqrt{17}}$

$\pm\frac{4}{\sqrt{17}} = 2 - 2\left(1 - \frac{8}{\sqrt{17}}\right) = \frac{16}{\sqrt{17}}$ false.

Therefore reject **case 2**.

Answer: $t = n\pi,\ n \in Z^+ \cup \{0\}$	**1 mark:** Answer. Explicit rejection of case 2.