Neap

VCE Specialist Mathematics Units 3&4

Question and Answer Booklet

2024 Trial Examination 2

Reading time: 15 minutes

Writing time: 2 hours

Student's Name: _____

Teacher's Name: _____

Approved materials

- One approved technology (calculator or software)
- One scientific calculator
- One bound reference

Materials supplied

- Question and Answer Booklet of 23 pages
- Formula Sheet
- Multiple-Choice Answer Sheet

Instructions

- Write **your name** and your **teacher's name** in the spaces above on this page.
- Follow the instructions on your Multiple-Choice Answer Sheet.
- At the end of the examination, place your Multiple-Choice Answer Sheet inside the front cover of this booklet.

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents pages

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2024 VCE Specialist Mathematics Units 3&4 Examination.

Section A – Multiple-choice questions

Instructions

- Answer **all** questions in pencil on the Multiple-Choice Answer Sheet.
- Choose the response that is **correct** for the question.
- A correct answer scores 1; an incorrect answer scores 0.
- Marks will **not** be deducted for incorrect answers.
- No marks will be given if more than one answer is completed for any question.
- Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
- Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1

Consider the following statement.

'If my plants are turning yellow, then they are not getting enough water.'

What is the contrapositive of this statement?

A. If they are getting enough water, then my plants will grow well.

B. If they are getting enough water, then my plants are not turning yellow.

C. If they are not getting enough water, then my plants are turning yellow.

D. If my plants are not turning yellow, then they are getting enough water.

Question 2

Let $f(x) = \dfrac{\sec(2x)}{\cos(x)}$.

How many asymptotes does the graph of f have in the interval $\left(-\pi, \dfrac{\pi}{2}\right)$?

A. 3

B. 4

C. 5

D. 6

Question 3

Let $f(x) = \sin^{-1}\left(\log_e(x)\right)$.

The gradient of f is decreasing for

A. $\frac{1}{e} < x < e^{\frac{\sqrt{5}+1}{2}}$

B. $\frac{1}{e} < x < e^{\frac{\sqrt{5}-1}{2}}$

C. $e < x < e^{\frac{\sqrt{5}-1}{2}}$

D. $\frac{1}{e} < x < e^{\frac{\sqrt{5}}{2}}$

Question 4

One of the roots of $2z^3 + bz^2 + cz + 100 = 0$ is $2 + 4i$, where b and c are real numbers.

The value of $b + c$ is

A. 17

B. 20

C. 25

D. 28

Question 5

On an Argand diagram, which one of the following points lies on the path defined by $|z + 3 + i| = |z - 3 - 3i|$?

A. (2, 3)

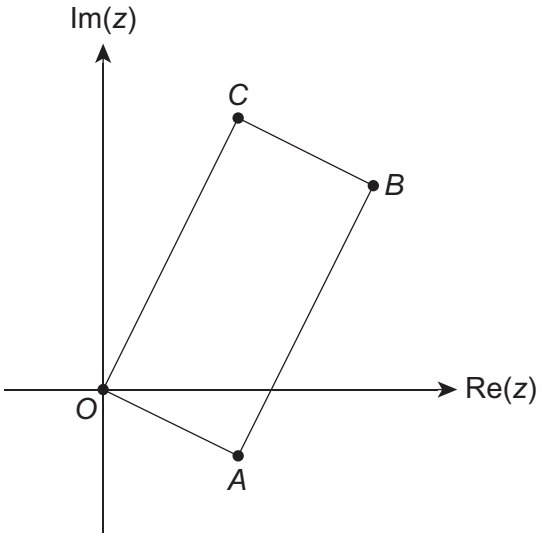
B. (–2, 4)

C. (2, –3)

D. (–1, 2)

Question 6

On the Argand diagram below, $OABC$ is a rectangle such that $OC = 2OA$. Point A is represented by the complex number z.



Point B is represented by the complex number

A. $z + 2iz$

B. $z + 1 + 2i$

C. $z + i$

D. $2z + iz$

Question 7

The length of the curve defined by the parametric equations $x = e^{2t}$ and $y = te^{t}$ for $0 \leq t \leq 1$ is given by

A. $\int_0^1 \sqrt{4e^{2t} + \left(e^t + te^t\right)^2}\, dt$

B. $\int_0^1 \sqrt{e^{4t} + \left(e^t + te^t\right)^2}\, dt$

C. $\int_0^1 \sqrt{4e^{4t} + \left(e^t + t\right)^2}\, dt$

D. $\int_0^1 \sqrt{4e^{4t} + \left(e^t + te^t\right)^2}\, dt$

Question 8

Using integration by parts, $\int x(x+1)^9\,dx$ can be expressed as

A. $\frac{1}{10}(x-1)x^{10} - \int \frac{1}{10}(x+1)^{10}\,dx$

B. $\frac{1}{10}x(x+1)^{10} + \int \frac{1}{10}(x+1)^9\,dx$

C. $\frac{1}{10}x(x+1)^{10} - \int \frac{1}{10}(x+1)^{10}\,dx$

D. $\frac{1}{10}x(x+1)^9 - \int \frac{1}{10}(x+1)^{10}\,dx$

Question 9

The following algorithm calculates the cross product of two vectors.

Inputs:

x_1, y_1, z_1 = vector 1

x_2, y_2, z_2 = vector 2

Define cross_product(vector 1, vector 2)

$x \leftarrow y_1 \times z_2 - z_1 \times y_2$

$y \leftarrow$ _____

$z \leftarrow x_1 \times y_2 - y_1 \times x_2$

result $\leftarrow (x, y, z)$

Return result

Which one of the following is the missing part of the algorithm?

A. $y_1 \times z_2 - z_1 \times y_2$

B. $x_1 \times z_2 - x_2 \times z_1$

C. $z_1 \times z_2 - x_1 \times x_2$

D. $z_1 \times x_2 - x_1 \times z_2$

Question 10

Consider the function $f(x) = \frac{dy}{dx} = 3x^2 + x$, where $y_0 = 0 = y(1)$.

Using Euler's formula with step size 0.1, y_3 is

A. $0.4 + 0.1f(1.3)$

B. $0.873 + f(1.2)$

C. $0.4 + 0.1f(1.2)$

D. $0.873 + 0.1f(1.2)$

Question 11

Consider the planes $\Pi_1 : 3x - 2y + z = 9$ and $\Pi_2 : -2x + 4y + 3z = -1$.

The angle between these planes is closest to

A. 43°

B. 46°

C. 51°

D. 57°

Question 12

A particle travelling in a straight line has velocity v at position x.

If the velocity of the particle is given by $v = \dfrac{2}{4 + x^2}$, then its acceleration is given by

A. $\tan^{-1}\left(\dfrac{x}{2}\right)$

B. $\dfrac{-8x}{\left(x^2 + 4\right)^3}$

C. $\tan^{-1}(2x)$

D. $\dfrac{-4x}{\left(x^2 + 4\right)^2}$

Question 13

A car starts from rest and accelerates uniformly at 2 m s^{-2} for five seconds. After this, the car accelerates constantly at 1 m s^{-2}.

The total time it takes for the car to travel 60 m is closest to

A. 8 s

B. 9 s

C. 10 s

D. 11 s

Question 14

A ladder of length 10 m rests against a vertical wall. The bottom of the ladder slides away from the wall at a rate of 1 m s^{-1}.

When the bottom of the ladder is 6 m from the wall, the top of the ladder will be sliding down the wall at a speed of

A. 0.5 m s^{-1}

B. 0.7 m s^{-1}

C. 0.75 m s^{-1}

D. 1 m s^{-1}

Question 15

Which one of the following statements is true for the vectors $\underset{\sim}{a} = \underset{\sim}{i} - 2\underset{\sim}{j} + 2\underset{\sim}{k}$, $\underset{\sim}{b} = -2\underset{\sim}{i} - 4\underset{\sim}{j} + 4\underset{\sim}{k}$ and $\underset{\sim}{c} = 4\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}$?

A. $\underset{\sim}{a}$ is parallel to $\underset{\sim}{b}$.

B. $\underset{\sim}{b} \perp \underset{\sim}{c}$

C. $|\underset{\sim}{a}| = |\underset{\sim}{b}|$

D. $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are linearly dependent.

Question 16

An object is thrown horizontally with a speed of v m s^{-1} from a height of h m and hits the ground after time of T s.

The value of T is given by

A. $\sqrt{\dfrac{2h}{g}}$

B. $\sqrt{\dfrac{h}{g}}$

C. $\sqrt{\dfrac{h}{2g}}$

D. $\sqrt{\dfrac{h}{vg}}$

Question 17

A line is perpendicular to the plane $3x - 7y + 11z = -2$ and passes through the point (2, –5, 0).

The equation of the line is

A. $\underset{\sim}{r}(t) = (2 - 3t)\underset{\sim}{i} - (5 - 7t)\underset{\sim}{j} + 11t\underset{\sim}{k}$

B. $\underset{\sim}{r}(t) = (-2 + 3t)\underset{\sim}{i} + (-5 + 7t)\underset{\sim}{j} + 11t\underset{\sim}{k}$

C. $\underset{\sim}{r}(t) = (2 + 3t)\underset{\sim}{i} - (5 + 7t)\underset{\sim}{j} + 11t\underset{\sim}{k}$

D. $\underset{\sim}{r}(t) = (3 + 2t)\underset{\sim}{i} - (7 + 5t)\underset{\sim}{j} + 11t\underset{\sim}{k}$

Question 18

A fruit farm grows apples and pears. The apples have a mean mass of 180 g with a variance of 15 g. The pears have a mean mass of 200 g with a variance of 10 g. The masses of the pears are independent of the masses of the apples.

What is the mean mass and standard deviation of a sample that contains four apples and three pears?

A. $\mu = 1320$ g, $\sigma = 30\sqrt{2}$ g

B. $\mu = 1340$ g, $\sigma = 3\sqrt{10}$ g

C. $\mu = 1340$ g, $\sigma = 30\sqrt{2}$ g

D. $\mu = 1320$ g, $\sigma = 3\sqrt{10}$ g

Question 19

The time, T, it takes to bake a batch of cookies is a function of its weight, w kg, where $T = 0.4w + 0.2$. The weights of the batches of cookies are normally distributed with a mean of 5 kg and a standard deviation of 0.05 kg.

When 50 batches of cookies are baked, a 95% confidence interval for the average time per batch is closest to

A. (2.19, 2.21)

B. (2.14, 2.26)

C. (2.16, 2.24)

D. (1.77, 2.63)

Question 20

A factory has two machines that make gears. Each machine operates independently, and the time it takes for each machine to make a gear is normally distributed with a mean of 20 minutes and a standard deviation of 2 minutes.

The probability that the time it takes for each machine to make one gear will differ by more than 1 minute is closest to

A. 0.6381

B. 0.6609

C. 0.7237

D. 0.7564

End of Section A

Section B

Instructions

- Answer **all** questions in the spaces provided.
- Write your responses in English.
- Unless otherwise specified, an **exact** answer is required to a question.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
- Take the **acceleration due to gravity** to have magnitude g m s^{-2}, where $g = 9.8$.

Question 1 (12 marks)

Consider the points $A(2, -1, 5)$, $B(0, 3, 2)$ and $C(-2, 1, 1)$.

a. Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$. 2 marks

b. Hence, or otherwise, explain why points A, B and C are not collinear. 1 mark

c. Find the area of the triangle ABC. 1 mark

d. Points A, B and C are contained in the plane Π_1.
Show that the Cartesian equation of the plane Π_1 is $-5x + 2y + 6z = 18$. 2 marks

A second plane, Π_2, has the Cartesian equation $x - 3y + z = -1$.
L is the line of intersection between the planes Π_1 and Π_2.

e. Find the equation of line L. 4 marks

f. Find the distance between point A and the plane Π_2. 2 marks

Question 2 (10 marks)

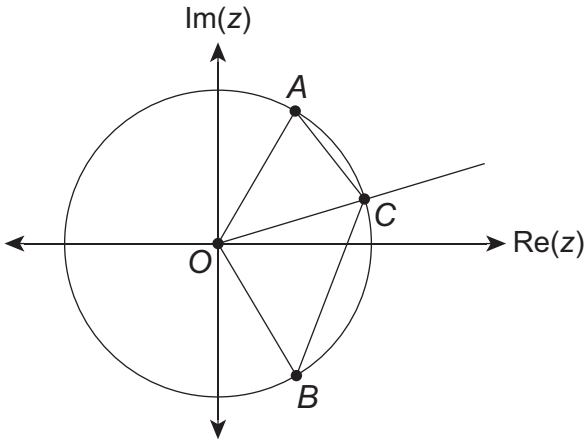
Let $u = \sqrt{2} + \sqrt{6}i$ and $v = \sqrt{2} - \sqrt{6}i$.

a. Write u and v in polar form. 2 marks

b. Show that u and v are roots of $z^3 + 16\sqrt{2} = 0$. 2 marks

c. Hence, factorise the polynomial $z^3 + 16\sqrt{2}$ in the set of real numbers. 2 marks

On the Argand diagram below, points *A* and *B* are represented by *u* and *v*, respectively.
It is given that $\angle AOC = \frac{2\pi}{9}$.



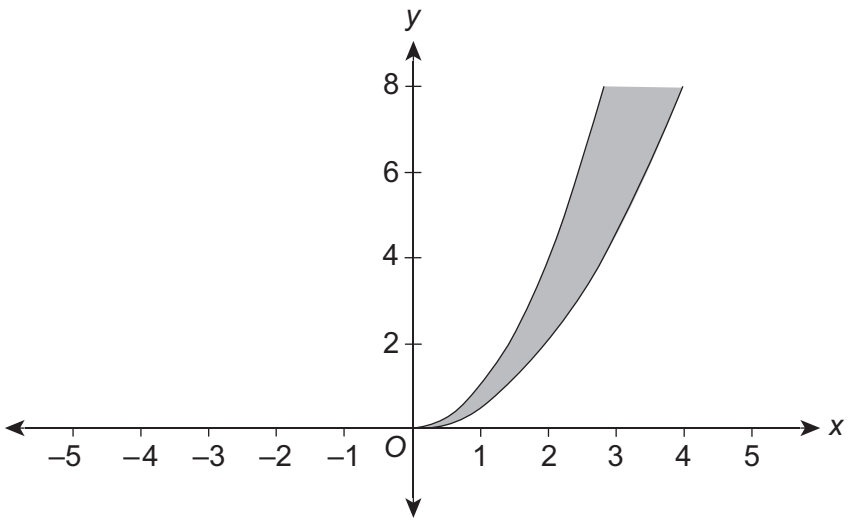
d. Show that $\angle COB = \frac{4\pi}{9}$. 1 mark

e. Write the equation of the ray passing through points O and C. 1 mark

f. Write the area of the quadrilateral $AOBC$ in the form $A\sin\alpha(1+B\cos\alpha)$, where A, B, $\alpha \in R$. 2 marks

Question 3 (11 marks)

Parts of the graphs of $y = x^2$ and $y = \frac{1}{2}x^2$ for $0 \leq y \leq 8$ are shown.



A vase is obtained by rotating the shaded region bounded by $y = x^2$, $y = \frac{1}{2}x^2$ and $y = 8$ about the y-axis. The vase's dimensions are in metres.

a. Find the **complete** surface area of the vase. Give your answer correct to two decimal places. 4 marks

The vase is filled with water to a depth of h m.

b. Write down a definite integral in terms of y and h for the volume of water in the vase. Hence, find an expression for the volume of water in terms of h. 2 marks

c. The vase is emptied and is refilled with water at a rate of $\sqrt{h}$ m^3 per minute.

i. How long will it take for the vase to completely fill with water? Give your answer in minutes, correct to one decimal place. 2 marks

ii. Express the volume of water in the vase as a function of time. Give your answer in the form $\sqrt[3]{\frac{pt^4}{q\pi}}$, where $p, q \in Z$. 3 marks

Question 4 (9 marks)

Let $I_n = \int_0^1 (1 - x^2)^{\frac{n}{2}} dx$, where $n \geq 0$ is an integer.

a. Find I_1. 1 mark

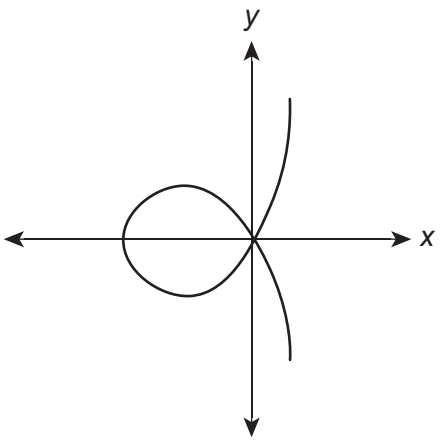
b. Use integration by parts to prove that $I_n = \frac{n}{n+1} I_{n-2}$ for all $n \geq 2$. 4 marks

c. Hence, find I_5. 2 marks

d. A solid of revolution is obtained by rotating $y = (1 - x^2)^{\frac{n}{2}}$ about the x-axis between $x = 0$ and $x = 1$. The volume of the solid can be expressed as pI_{qn}.
State the values of p and q. 2 marks

Question 5 (8 marks)

The graph below shows a curve with the parametric equations $x = (\sec t + a\cos t)\cos t$ and $y = (\sec t + a\cos t)\sin t$, where $a < -1$ and $t \geq 0$.



a. Verify that the Cartesian equation of the curve is $(x - 1)(x^2 + y^2) = ax^2$. 3 marks

b. State the equation of the asymptote. 1 mark

Let $a = -2$.

c. Find the coordinates of the maximum and minimum points, correct to two decimal places. 2 marks

d. Write a definite integral that represents the arc length of the closed region of the curve. Evaluate the integral, correct to two decimal places. 2 marks

Question 6 (10 marks)

A school bus has a maximum load capacity of 3000 kg. The weights of the students who ride the bus follow a normal distribution with a mean of 70 kg and a standard deviation of 15 kg.

On a particular morning, 42 students use the bus.

a. What is the probability that the total weight of the students on the bus will exceed 3000 kg? Give your answer correct to four decimal places. 1 mark

On another morning, there are n students on the bus.

b. Determine the maximum value of n such that there is a less than 1% chance of the bus exceeding the maximum load capacity. 1 mark

The bus picks up students from six bus stops on the way to school. At each bus stop, the bus waits for a mean time of 2 minutes with a variance of 24 seconds. The times spent waiting at each stop are normally distributed and independent of each other. The total travel time, including the time spent waiting at bus stops, follows a normal distribution with a mean of 55 minutes and a variance of 3 minutes.

c. What is the mean and standard deviation of the travel time **excluding** the time spent waiting at bus stops? 2 marks

d. What is the probability that the mean travel time **excluding** the time spent waiting at bus stops is less than 41 minutes for five consecutive days? Give your answer correct to four decimal places. 1 mark

For the first 10 days of a new school year, the mean travel time was 57 minutes with a standard deviation of 4 minutes. The school's management claims that the bus is taking longer than 55 minutes to arrive at school.

A test at the 5% significance level is to be conducted to support this claim.

e. Write down the null hypothesis, H_0, and the alternative hypothesis, H_1, for this test. 1 mark

f. State the *p*-value, correct to four decimal places. Hence, draw a conclusion about the null hypothesis in **part e.** 2 marks

g. The critical sample mean is the smallest sample mean for which H_0 will not be rejected.

What is the critical sample mean in this test? Give your answer in minutes, correct to three decimal places. 2 marks

End of examination questions

