

The Mathematical Association of Victoria

Trial Examination 2024

**SPECIALIST MATHEMATICS**

**Written Examination 2 - SOLUTIONS**

**SECTION A: Multiple Choice**

Question	Answer	Question	Answer
1	B	11	A
2	B	12	B
3	C	13	C
4	D	14	B
5	B	15	A
6	B	16	B
7	A	17	B
8	D	18	D
9	B	19	A
10	D	20	A

**Question 1**                      **Answer B**

$2xy - y^2 = -3$ ,  $x = 1$  for  $y > 0$  gives  $(1, 3)$

gradient to the curve  $= \frac{3}{2}$

$2xy - y^2 = -3 \mid x=1$   
 $-y^2 + 2 \cdot y = -3$   
 $\text{solve}(-y^2 + 2 \cdot y = -3, y)$   
 $\{y = -1, y = 3\}$   
 $\text{impDiff}(2 \cdot x \cdot y - y^2 = -3, x, y)$   
 $y' = \frac{-2 \cdot y}{2 \cdot x - 2 \cdot y}$   
 $y' = \frac{-2 \cdot 3}{2 \cdot 1 - 2 \cdot 3}$   
 $y' = \frac{3}{2}$

$\text{solve}(2 \cdot x \cdot y - y^2 = -3 \text{ and } x = 1 \text{ and } y > 0, x, y)$   
 $x = 1 \text{ and } y = 3$   
 $\text{impDiff}(2 \cdot x \cdot y - y^2 = -3, x, y)$                        $\frac{-y}{x - y}$   
 $\frac{-y}{x - y} \mid x = 1 \text{ and } y = 3$                        $\frac{3}{2}$

**Question 2 Answer B**

$$y = \frac{ax^3}{x^2 + bx - 2}$$

two vertical asymptotes at  $x = 1$  and  $x = -2$ , oblique asymptote at  $y = 3x - 3$ .

Possible graph:  $y = 3x - 3 + \frac{ax + c}{(x + 2)(x - 1)} = 3x - 3 + \frac{ax + c}{x^2 + x - 2}$  where  $c$  is a real constant.

Gives possible values  $a = 3, b = 1$

The screenshot shows a CAS interface with the following steps and results:

- Input:  $\text{combine}\left(3x - 3 + \frac{ax + c}{(x + 2)(x - 1)}\right)$
- Result:  $\frac{3 \cdot x^3 + a \cdot x - 9 \cdot x + c + 6}{(x + 2) \cdot (x - 1)}$
- Input:  $\text{expand}\left((x + 2) \cdot (x - 1)\right)$
- Result:  $x^2 + x - 2$
- Input:  $\text{comDenom}\left(3 \cdot x - 3 + \frac{a \cdot x + c}{(x - 2) \cdot (x - 1)}\right)$
- Result:  $\frac{3 \cdot x^3 - 12 \cdot x^2 + a \cdot x + 15 \cdot x + c - 6}{x^2 - 3 \cdot x + 2}$

**Question 3 Answer C**

$$y = a \operatorname{cosec}\left(\frac{\pi}{2}x + \pi\right) = \frac{a}{\sin\left(\frac{\pi}{2}x + \pi\right)}$$

$$\text{Period} = \frac{2\pi}{\frac{\pi}{2}} = 4$$

So for cosec graph, asymptotes will be 2 units apart.

In the interval  $-3 < x < 3$

Asymptotes at  $x = -2, x = 0, x = 2$

The screenshot shows a CAS interface with the following elements:

- Input:  $\frac{a}{\sin\left(\frac{\pi}{2}x + \pi\right)}$
- Result:  $\frac{-a}{\sin\left(\frac{x \cdot \pi}{2}\right)}$
- Graph: A plot of the function  $y = \frac{-a}{\sin\left(\frac{x \cdot \pi}{2}\right)}$  on a coordinate plane. The x-axis ranges from -3 to 3, and the y-axis ranges from -4 to 4. Vertical asymptotes are shown at  $x = -2, 0, 2$ . The graph consists of three U-shaped branches opening upwards and three inverted U-shaped branches opening downwards.
- Input:  $\text{domain}\left(a \cdot \operatorname{csc}\left(\frac{\pi}{2} \cdot x + \pi\right), x\right) \mid -3 < x < 3$
- Result:  $x \neq -2$  and  $x \neq 0$  and  $x \neq 2$  and  $-3 < x < 3$

**Question 4**                      **Answer D**

Let  $z = x + yi$

$$\therefore \frac{z\bar{z}}{|z|} = \frac{(x + yi)(x - yi)}{|(x + yi)|} = \sqrt{13}$$


$$\frac{x^2 + y^2}{\sqrt{x^2 + y^2}} = \sqrt{13}$$

gives  $\sqrt{x^2 + y^2} = \sqrt{13}$

$$x^2 + y^2 = 13$$

Possible  $z = 2^2 + 3^2$  gives  $z = 2 + 3i$

$$iz = i2 + 3i^2 = -3 + 2i$$



$$\frac{z \cdot \text{conj}(z)}{|z|} = \sqrt{13} \quad |z = x + yi \cdot i| \quad \sqrt{x^2 + y^2} = \sqrt{13}$$

**Question 5**                      **Answer B**

$x(t) = (t + 1)^3$  and  $y(t) = \frac{2}{t + 1}$  where  $t \geq 0$ .

$$x'(t) = 3(t + 1)^2$$

$$y'(t) = \frac{-2}{(t + 1)^2}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = \frac{-2}{(t + 1)^2} \times \frac{1}{3(t + 1)^2} = \frac{-2}{3(t + 1)^4}$$

Gradient of tangent at  $t = 1$  is  $\frac{-2}{3 \times 16} = -\frac{1}{24}$

perpendicular to the tangent at  $t = 1$ , gradient = 24

Edit Action Interactive ✕

0.5 1 2
↶ ↷
∫ dx ∫ dx
Simp
f dx
⌵
⌶
▶

```
define x(t)=(t+1)3
done

define y(t)=2/t+1
done

d
dt
(x(t))
3·(t+1)2

d
dt
(y(t))
-2
(t+1)2
```

$\frac{\frac{d}{dt}(y(t))}{\frac{d}{dt}(x(t))} \Big _{t=1}$	$-\frac{1}{24}$	<table border="1" style="border-collapse: collapse; width: 100%;"> <tr> <td style="padding: 5px;"> <math>\frac{d}{dt}\left(\frac{2}{t+1}\right) \cdot \left(\frac{d}{dt}\left((t+1)^3\right)\right)^{-1}</math> </td> <td style="padding: 5px; text-align: center;"> <math>\frac{-2}{3 \cdot (t+1)^4}</math> </td> </tr> <tr> <td style="padding: 5px;"> <math>\frac{d}{dt}\left(\frac{2}{t+1}\right) \cdot \left(\frac{d}{dt}\left((t+1)^3\right)\right)^{-1} \Big _{t=1}</math> </td> <td style="padding: 5px; text-align: center;"> <math>\frac{-1}{24}</math> </td> </tr> </table>	$\frac{d}{dt}\left(\frac{2}{t+1}\right) \cdot \left(\frac{d}{dt}\left((t+1)^3\right)\right)^{-1}$	$\frac{-2}{3 \cdot (t+1)^4}$	$\frac{d}{dt}\left(\frac{2}{t+1}\right) \cdot \left(\frac{d}{dt}\left((t+1)^3\right)\right)^{-1} \Big _{t=1}$	$\frac{-1}{24}$
$\frac{d}{dt}\left(\frac{2}{t+1}\right) \cdot \left(\frac{d}{dt}\left((t+1)^3\right)\right)^{-1}$	$\frac{-2}{3 \cdot (t+1)^4}$					
$\frac{d}{dt}\left(\frac{2}{t+1}\right) \cdot \left(\frac{d}{dt}\left((t+1)^3\right)\right)^{-1} \Big _{t=1}$	$\frac{-1}{24}$					

**Question 6 Answer B**

The pseudocode ultimately is calculating  $\sin^2(1^\circ) + \sin^2(2^\circ) + \sin^2(3^\circ) + \dots + \sin^2(89^\circ)$  or  $\sum_{t=1}^{89} \sin^2(t^\circ)$ .

CAS can be used to calculate the answer.

Or otherwise note that  $\sin^2 x + \cos^2 x = 1$  and  $\sin^2(90^\circ - n) = \cos^2(n)$ .

This means  $\sin^2 1^\circ + \sin^2 89^\circ = 1$  where there are 44 pairs in this algorithm.

$44 + \sin^2 45^\circ = 44.5$ .

$\sum_{t=1}^{89} (\sin(t^\circ))^2$	$\frac{89}{2}$
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When convert the pseudocode into Python,

<pre> 1.1 MAV2024.py 8/8 from math import * a = 0 t = 1 while t &lt; 90:     a += sin(radians(t)) ** 2     t += 1 print(a) </pre>	<table border="1" style="border-collapse: collapse; width: 100%;"> <tr> <td style="padding: 5px;"> <pre> 1.2 Python Shell 4/4 &gt;&gt;&gt;#Running MAV2024.py &gt;&gt;&gt;from MAV2024 import * 44.5 &gt;&gt;&gt;  </pre> </td> </tr> </table>	<pre> 1.2 Python Shell 4/4 &gt;&gt;&gt;#Running MAV2024.py &gt;&gt;&gt;from MAV2024 import * 44.5 &gt;&gt;&gt;  </pre>
<pre> 1.2 Python Shell 4/4 &gt;&gt;&gt;#Running MAV2024.py &gt;&gt;&gt;from MAV2024 import * 44.5 &gt;&gt;&gt;  </pre>		

**Question 7 Answer A**

$$\int_{-1}^2 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

$$= \int_{-1}^2 \sqrt{\left(\frac{d}{dt}(2t)\right)^2 + \left(\frac{d}{dt}\left((t-1) - (1-2t)^2\right)\right)^2} dt$$

$$= \int_{-1}^2 \sqrt{(2)^2 + (5-8t)^2} dt$$

$$= 19.617 \approx 20$$

$\int_{-1}^2 \text{norm}\left(\frac{d}{dt}\left(\left[t-1-(1-2\cdot t)^2\right]\right)\right) dt$	19.6175
$y(t):=t-1-(1-2\cdot t)^2$	Done
$\text{arcLen}\left(y\left(\frac{t}{2}\right),t,-1\cdot 2,2\cdot 2\right)$	19.6175

**Question 8**                      **Answer D**

$$a = v(1+v)^2$$

$$v \frac{dv}{dx} = v(1+v)^2$$

$$\frac{dv}{dx} = (1+v)^2 \text{ where } v \neq 0$$

$$\frac{dx}{dv} = \frac{1}{(1+v)^2}$$

$$x = \int (1+v)^{-2} dv$$

$$x = -\frac{1}{1+v} + c$$

Given  $x = 1, v = 1, 1 = -\frac{1}{1+1} + c \Rightarrow c = \frac{3}{2}$

$$x = -\frac{1}{1+v} + \frac{3}{2}$$

At  $v = 10, x = -\frac{1}{1+10} + \frac{3}{2}$

At  $v = 10, x = \frac{31}{22}$  metres

deSolve( $v \cdot v' = v \cdot (1+v)^2$ and $v(1)=1,x,v$ )	
	$v = \frac{-2}{2 \cdot x - 3} - 1$
$\Delta$ solve( $v = \frac{-2}{2 \cdot x - 3} - 1, x$ )   $v = 10$	$x = \frac{31}{22}$

**Question 9**                      **Answer B**

$$\underline{a} = m\underline{i} + 4\underline{j} + 5\underline{k}, \underline{b} = -\underline{i} - n\underline{j} \text{ and } \underline{c} = 2\underline{i} + p\underline{k}$$

For linearly **dependent**, allow  $\underline{a} = E\underline{b} + F\underline{c}$  where  $E, F$  are real constants

$$m\underline{i} + 4\underline{j} + 5\underline{k} = E(-\underline{i} - n\underline{j} + 0\underline{k}) + F(2\underline{i} + 0\underline{j} + p\underline{k})$$

	$\underline{i}$	$\underline{j}$	$\underline{k}$
$\underline{a}$	$m$	4	5
$\underline{b}$	$-E$	$-En$	$0E$
$\underline{c}$	$2F$	$0F$	$pF$

For  $\underline{a} = E\underline{b} + F\underline{c}$  we get the following equations

$$\begin{cases} m = -E + 2F \\ 4 = -En + 0F \\ 5 = 0E + pF \end{cases}$$

$$\begin{cases} m = -E + 2F \\ 4 = -En \Rightarrow E = -\frac{4}{n} \\ 5 = pF \Rightarrow F = \frac{5}{p} \end{cases}$$

Giving  $m = -\left(-\frac{4}{n}\right) + 2\left(\frac{5}{p}\right)$

$$m = \left(\frac{4}{n}\right) + 2\left(\frac{5}{p}\right)$$

$$\therefore m = \frac{4}{n} + \frac{10}{p}$$

The image shows a handwritten derivation. It starts with the equation  $\text{solve}\left(\det\begin{pmatrix} m & -1 & 2 \\ 4 & -n & 0 \\ 5 & 0 & p \end{pmatrix} = 0, m\right)$ . This leads to the expression  $m = \frac{2 \cdot (5 \cdot n + 2 \cdot p)}{n \cdot p}$ . A subsequent step shows the expansion of this fraction:  $\text{expand}\left(m = \frac{2 \cdot (5 \cdot n + 2 \cdot p)}{n \cdot p}\right)$  resulting in  $m = \frac{4}{n} + \frac{10}{p}$ .

**Question 10**      **Answer D**

$$\dot{\underline{r}}(t) = \sin(t)\cos(t)\underline{i} + \cos(2t)\underline{j}$$

$$\underline{r}(t) = \int (\sin(t)\cos(t)\underline{i} + \cos(2t)\underline{j}) dt$$

$$\underline{r}(t) = \int \left(\frac{1}{2}\sin(2t)\underline{i} + \cos(2t)\underline{j}\right) dt$$

$$\therefore \underline{r}(t) = -\frac{1}{4}\cos(2t)\underline{i} + \frac{1}{2}\sin(2t)\underline{j} + \underline{c}$$

Given  $\underline{r}(\pi) = 2\underline{i} - 3\underline{j}$

$$2\hat{i} - 3\hat{j} = -\frac{1}{4}\cos(2\pi)\hat{i} + \frac{1}{2}\sin(2\pi) + \underline{c}$$

$$\underline{c} = \frac{1}{4}\cos(2\pi)\hat{i} - \frac{1}{2}\sin(2\pi) + 2\hat{i} - 3\hat{j}$$

$$\underline{c} = \frac{1}{4}\hat{i} + 2\hat{i} - 3\hat{j} = \frac{9}{4}\hat{i} - 3\hat{j}$$

Gives

$$\underline{r}(t) = -\frac{1}{4}\cos(2t)\hat{i} + \frac{1}{2}\sin(2t) + \frac{9}{4}\hat{i} - 3\hat{j}$$

displacement of the body at time  $t$ ,  $\underline{r}(t)$  is given by

$$\underline{r}(t) = \left(-\frac{1}{4}\cos(2t) + \frac{9}{4}\right)\hat{i} + \left(\frac{1}{2}\sin(2t) - 3\right)\hat{j}$$

$$\int_{\pi}^t \begin{bmatrix} \sin(t) \cdot \cos(t) \\ \cos(2 \cdot t) \end{bmatrix} dt + \begin{bmatrix} 2 \\ -3 \end{bmatrix} = \begin{bmatrix} \frac{9}{4} - \frac{\cos(2 \cdot t)}{4} \\ \frac{\sin(2 \cdot t)}{2} - 3 \end{bmatrix}$$

**Question 11**      **Answer A**

$\frac{dP}{dt} = P\left(6 - \frac{P}{8000}\right)$  with initial population,  $P$ , of 4000 bacteria.

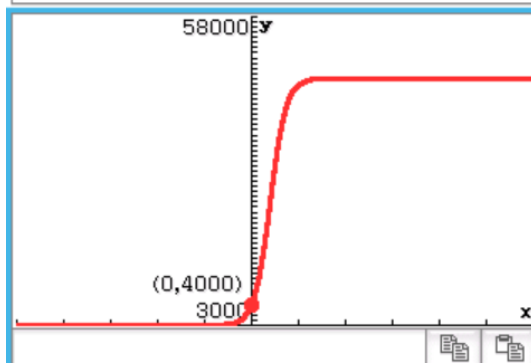
Solution to DE is  $P = \frac{48000(-\frac{1}{11})e^{6t}}{-\frac{1}{11}e^{6t} - 1}$

Giving horizontal asymptote  $P = 48000$

$$\text{propFrac}\left(\frac{48000 \cdot (-\frac{1}{11}) \cdot e^{6 \cdot x}}{-\frac{1}{11} \cdot e^{6 \cdot x} - 1}\right)$$

$$\frac{48000 \cdot e^{6 \cdot x}}{e^{6 \cdot x} + 11}$$

solve(f(x)=48000, x)  
No Solution



The image shows three screenshots of a CAS calculator interface, likely TI-Nspire CX, used to solve a differential equation and find the value of a parameter.

**First Screenshot:** Shows the differential equation  $y' = y \cdot \left(6 - \frac{y}{8000}\right)$  being solved. The solution is given as  $\left\{ \frac{\frac{1}{(|y|)^{48000}}}{\frac{1}{(|y-48000|)^{48000}}} = e^{\frac{x}{8000}} \cdot \text{const}(1) \right\}$ . Below this, the equation is rearranged to  $\text{solve}\left(\left(\frac{y}{y-48000}\right)^{\frac{1}{48000}} = a \cdot e^{\frac{x}{8000}}, y\right)$ .

**Second Screenshot:** Shows the definition of a function  $f(x) = \frac{48000 \cdot a^{48000} \cdot e^{6 \cdot x}}{a^{48000} \cdot e^{6 \cdot x} - 1}$ . The value of  $a$  is determined by solving  $f(0) = 4000$ , resulting in  $\left\{ a^{48000} = -\frac{1}{11} \right\}$ .

**Third Screenshot:** Shows the function  $f(x) = \frac{48000 \cdot \left(-\frac{1}{11}\right) \cdot e^{6 \cdot x}}{-\frac{1}{11} \cdot e^{6 \cdot x} - 1}$ . The value of  $x$  is found by solving  $f(x) = 1/11$ , resulting in  $\left\{ x = \frac{-\ln(527999)}{6} + \frac{\ln(11)}{6} \right\}$ .

**Question 12                  Answer B**

Option A is incorrect as 3 is rational but  $\sqrt{3}$  is not;  
 Option C is incorrect as if  $x = 0$  and  $y = \sqrt{3}$  gives  $xy = 0$  where it is rational;  
 Option D is incorrect as 0 is even and  $0(0+1)$  is also even.

Proof of Option B:

*If an integer  $n$  is odd then  $n^2 + 2$  is odd.*

Proof:

Let  $n = 2\ell + 1$  where  $\ell \in \mathbb{Z}$ .

$$\begin{aligned} n^2 + 2 &= (2\ell + 1)^2 + 2 \\ &= 4\ell^2 + 1 + 2 \times 2\ell + 2 \\ &= 2(2\ell^2 + 2\ell + 1) + 1 \end{aligned}$$

Thus  $n^2 + 2$  is odd.



If  $n^2 + 2$  is odd then  $n$  is odd where  $n$  is an integer.

Proof:

We proceed by proving the contrapositive is true. The contrapositive of the above statement is

If  $n$  is even, then  $n^2 + 2$  is even where  $n$  is an integer.

Let  $n = 2\ell$  where  $\ell \in \mathbb{Z}$ .

$$\begin{aligned} n^2 + 2 &= (2\ell)^2 + 2 \\ &= 4\ell^2 + 2 \\ &= 2(2\ell^2 + 1) \end{aligned}$$

Thus  $n^2 + 2$  is even.

Therefore, if  $n^2 + 2$  is odd then  $n$  is odd where  $n$  is an integer.

**Question 13**                      **Answer C**

$$\begin{aligned} A &= |\underline{a} \times \underline{c}| \\ &= \left| \underline{a} \times (\sqrt{3}\underline{a} + \sqrt{2}\underline{b}) \right| \\ &= \left| \underline{a} \times \sqrt{3}\underline{a} + \underline{a} \times \sqrt{2}\underline{b} \right| \\ &= \left| \sqrt{3}\underline{a} \times \underline{a} + \sqrt{2}\underline{a} \times \underline{b} \right| \\ &= \left| \sqrt{3} \times 0 + \sqrt{2}\underline{a} \times \underline{b} \right| \\ &= \sqrt{2} \left| \underline{a} \cdot \underline{b} \cdot \sin(\theta) \cdot \hat{n} \right| \end{aligned}$$

From  $\underline{a} \cdot \underline{b} = 3$ , we know that  $|\underline{a}| \cdot |\underline{b}| \cdot \cos(\theta) = 3 \Rightarrow |\underline{a}| \cdot |\underline{b}| = \frac{3}{\cos(\theta)}$ .

$$\begin{aligned} A &= |\underline{a} \times \underline{c}| = \sqrt{2} \left| \underline{a} \cdot \underline{b} \cdot \sin(\theta) \right| \\ &= \sqrt{2} \times \left| \frac{3}{\cos(\theta)} \times \sin(\theta) \right| \\ &= \sqrt{2} \times \left| 3 \times \tan(\theta) \right| \\ &= \sqrt{2} \times \left| 3 \times \frac{1}{3} \right| \\ &= \sqrt{2} \end{aligned}$$

**Question 14**                      **Answer B**

$$\begin{aligned} \underline{b} \times (\underline{a} + \underline{b} + \underline{c}) &= \underline{b} \times \underline{0} \\ \underline{b} \times \underline{a} + \underline{b} \times \underline{b} + \underline{b} \times \underline{c} &= \underline{0} \\ -\underline{a} \times \underline{b} + \underline{0} + \underline{b} \times \underline{c} &= \underline{0} \\ \underline{a} \times \underline{b} &= \underline{b} \times \underline{c} \end{aligned}$$

**Question 15**                      **Answer A**

$$\begin{aligned} a(t) = e^{-2t} &\Rightarrow t = \frac{-1}{2} \log_e a \\ v(t) = -\frac{1}{2}e^{-2t} + 3 &\text{ and } x(t) = \frac{1}{4}e^{-2t} + 3t + 4 \end{aligned}$$

$$v = \frac{-1}{2} \cdot e^{-2 \cdot t + 3} \Big|_{t=0}^{-\ln(a)} \qquad v = 3 - \frac{a}{2}$$

**Question 16**      **Answer B**

If two planes are perpendicular to each other then  $a_1a_2 + b_1b_2 + c_1c_2 = 0$ .

$$2 \times 1 + 2 \times 1 + \lambda \times 1 = 0 \Rightarrow \lambda = -4$$

**Question 17**      **Answer B**

$$A_x = 2 \cdot \pi \cdot \int_0^a \left( \frac{x}{5} \cdot \sqrt{1 + \left( \frac{d}{dx} \left( \frac{x}{5} \right) \right)^2} \right) dx = \frac{a^2 \cdot \pi \cdot \sqrt{26}}{25}$$

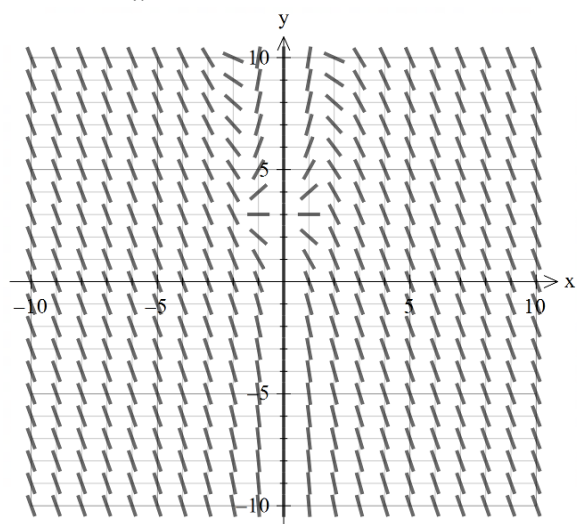
$$A_y = 2 \cdot \pi \cdot \int_0^{a/5} \left( 5 \cdot y \cdot \sqrt{1 + \left( \frac{d}{dy} (5 \cdot y) \right)^2} \right) dy = \frac{a^2 \pi \sqrt{26}}{5}$$

$$\frac{A_x}{A_y} = \frac{a^2 \cdot \pi \cdot \sqrt{26}}{25} \times \frac{5}{a^2 \pi \sqrt{26}} = \frac{1}{5}$$

**Question 18**      **Answer D**

Fourth quadrant means  $x$  value is positive and  $y$  value is negative.

Therefore  $\frac{y}{x^2}$  is negative and  $-3$  will make it more negative.



**Question 19 Answer A**

$$X - Y \sim N(0, 2\sigma^2)$$

$$P(-1 < X - Y < 1) = P\left(\frac{-1}{\sqrt{2}\sigma} < \frac{X - Y}{\sqrt{2}\sigma} < \frac{1}{\sqrt{2}\sigma}\right)$$

Therefore, the probability is independent from  $\mu$  but dependent on  $\sigma$ .

**Question 20 Answer A**

zInterval  $\sqrt{2}$ , 15, 36, 0.99: *stat.results*

"Title"	"z Interval"
"CLower"	14.3929
"CUpper"	15.6071
"x̄"	15.
"ME"	0.607129
"n"	36.
"σ"	1.41421

C-Level: 0.99

σ: 2<sup>0.5</sup>

x̄: 15

n: 36

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OneSampleZInt

Lower: 14.392871

Upper: 15.607129

x̄: 15

n: 36

<< Back     Help

OneSampleZInt

**END OF MULTIPLE CHOICE SOLUTIONS**

**SECTION B**

**Question 1 (10 marks)**

$$f(x) = \frac{ax^2 + 1}{x^2 - 3x + 2} \text{ where } a \in \mathbb{R} \setminus \{0\}.$$

a.  $f(x) = \frac{ax^2 + 1}{x^2 - 3x + 2} = \frac{ax^2 + 1}{(x-1)(x-2)}$

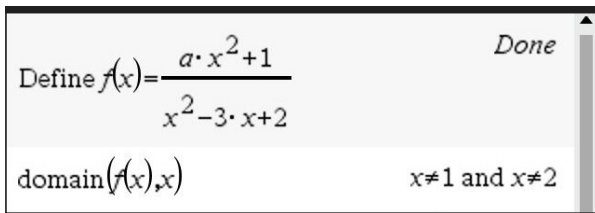
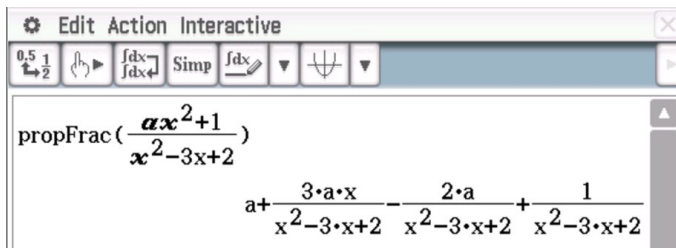
Equations of the vertical asymptotes.

$$x = 1, x = 2$$

Equation of the horizontal asymptote.

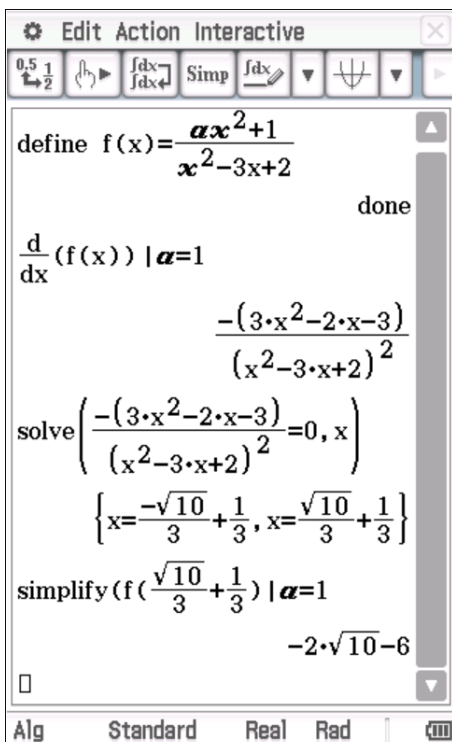
$$y = a$$

1A



b. Let  $a = 1$ . Local maximum stationary point  $\left(\frac{1 + \sqrt{10}}{3}, -2\sqrt{10} - 6\right)$

1A



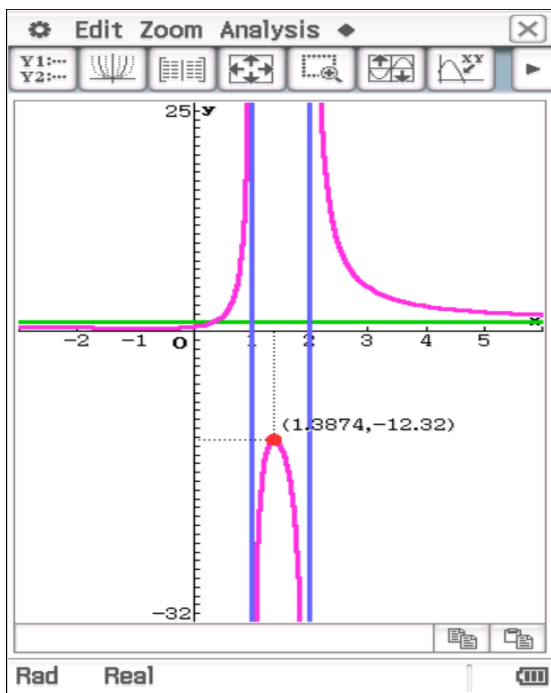
Define  $f'(x) = \frac{d}{dx}(f(x))$  Done

Define  $f''(x) = \frac{d^2}{dx^2}(f(x))$  Done

solve( $f'(x)=0$  and  $f''(x)<0, x$ )| $a=1$

$x = \frac{\sqrt{10} + 1}{3}$   $f(x)|_{x = \frac{\sqrt{10} + 1}{3}}$  and  $a=1$   $-2 \cdot (\sqrt{10} + 3)$

c. Let  $a = 1$ .



asymptotes  $x = 1, x = 2, y = 1$  3A

d. i. Stationary points

$$x = \frac{2a - 1 \pm \sqrt{4a^2 + 5a + 1}}{3a}$$

1A

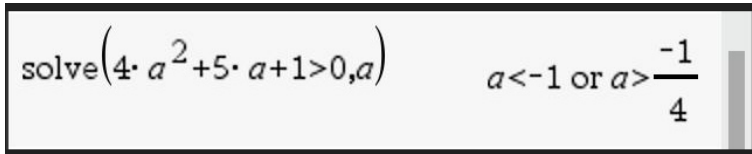
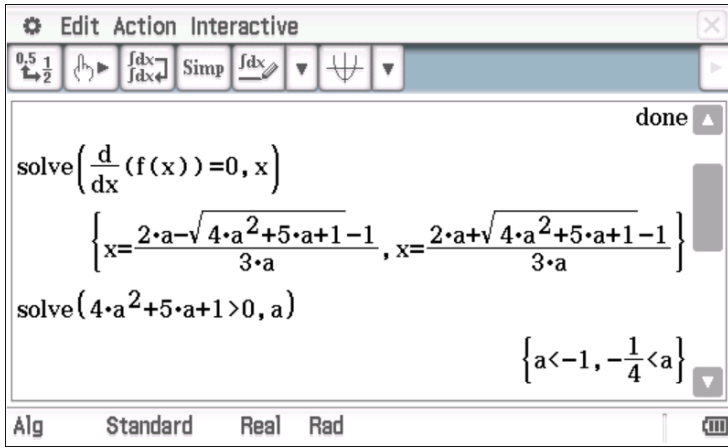
zeros( $f'(x), x$ )

$\left\{ \frac{-\left(\sqrt{4 \cdot a^2 + 5 \cdot a + 1} - 2 \cdot a + 1\right)}{3 \cdot a}, \frac{\sqrt{4 \cdot a^2 + 5 \cdot a + 1}}{3 \cdot a} \right\}$

ii. Two stationary points for  $4a^2 + 5a + 1 > 0$

$$a \in (-\infty, -1) \cup \left(-\frac{1}{4}, \infty\right)$$

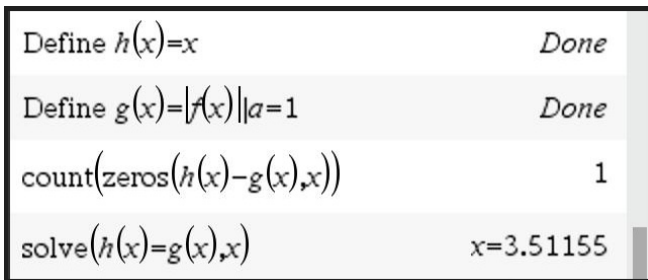
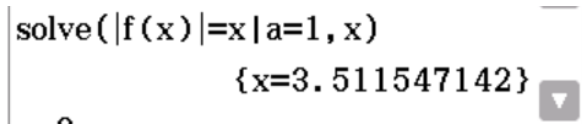
1A



$h(x) = x$  and  $g(x) = |f(x)|$  where  $a = 1$ .

e.  $h$  and  $g$  intersect once at  $x = 3.5115\dots$

1A



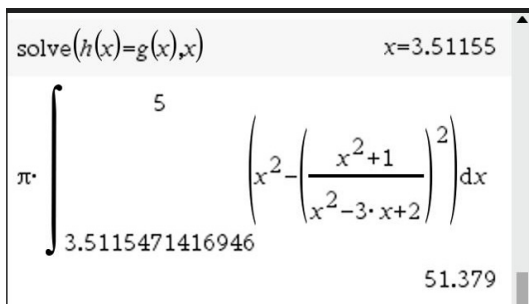
f. The region bounded by the curves of  $h$  and  $g$  and the line  $x = 5$  is rotated around the  $x$ -axis.

$$\text{Vol} = \pi \int_{3.5115\dots}^5 (x^2 - (f(x))^2) dx$$

$$\text{Vol} = \pi \int_{3.5115\dots}^5 \left(x^2 - \left(\frac{x^2 + 1}{x^2 - 3x + 2}\right)^2\right) dx$$

1M

Volume = 51.38 cubic units, to two decimal places 1A



**Question 2 (10 marks)**

$$\frac{dx}{dt} = 2 \operatorname{cosec}(x) \sin^2(2t) \text{ where } x = \pi \text{ when } t = \frac{\pi}{4}, t \geq 0$$

a. Use  $\sin^2(2t) = \frac{1}{2}(1 - \cos(4t))$

i.  $\frac{dx}{dt} = 2 \operatorname{cosec}(x) \sin^2(2t)$

$$\frac{dx}{dt} = 2 \operatorname{cosec}(x) \left( \frac{1}{2}(1 - \cos(4t)) \right)$$

$$\int \frac{1}{2 \operatorname{cosec}(x)} dx = \int \left( \frac{1}{2}(1 - \cos(4t)) \right) dt$$

$$\int \frac{\sin(x)}{2} dx = \frac{1}{2} \int (1 - \cos(4t)) dt$$

$$\int \sin(x) dx = \int (1 - \cos(4t)) dt \text{ of the required form } \int f(x) dx = \int g(t) dt$$

**1A**

ii. Solve in the form  $x = \cos^{-1}(at + b \sin(4t) + c)$

$$\int \sin(x) dx = \int (1 - \cos(4t)) dt$$

$$-\cos(x) = t - \frac{1}{4} \sin(4t) + c$$

**1M**

$$\cos(x) = -t + \frac{1}{4} \sin(4t) + c$$

$$\cos(\pi) = -\frac{\pi}{4} + \frac{1}{4} \sin\left(4 \cdot \frac{\pi}{4}\right) + c$$

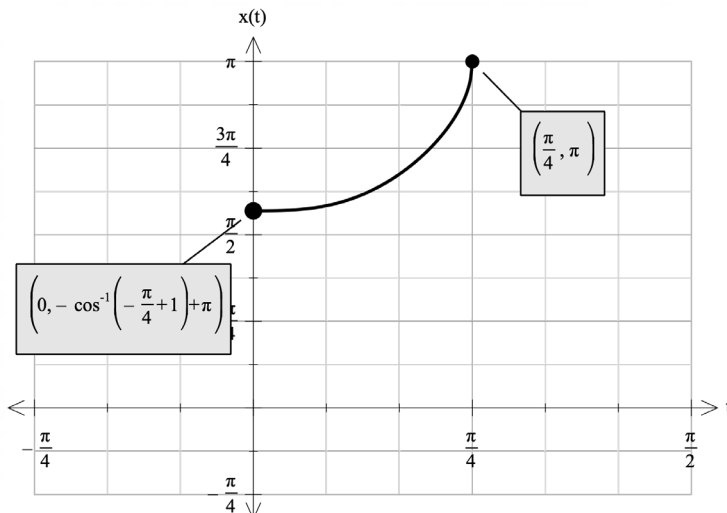
$$-1 = -\frac{\pi}{4} + c \therefore c = -1 + \frac{\pi}{4}$$

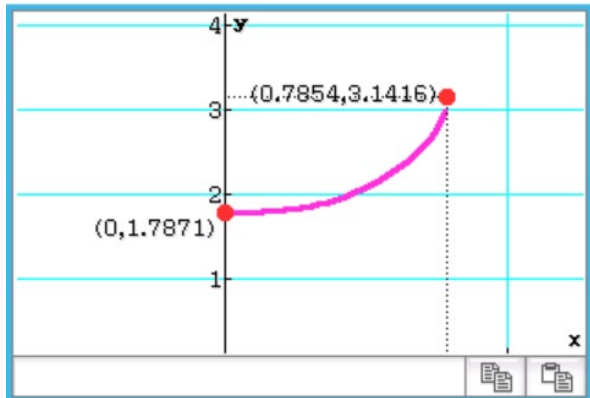
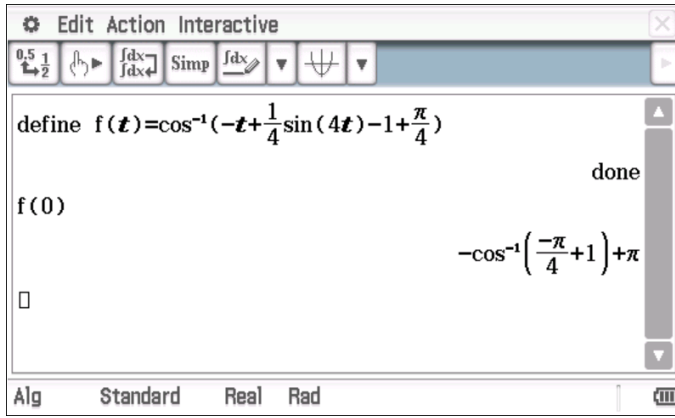
**1M**

$$x = \cos^{-1}\left(-t + \frac{1}{4} \sin(4t) - 1 + \frac{\pi}{4}\right)$$

**1A**

iii.





Domain  $\left[0, \frac{\pi}{4}\right]$

Range  $\left[\pi - \cos^{-1}\left(1 - \frac{\pi}{4}\right), \pi\right]$

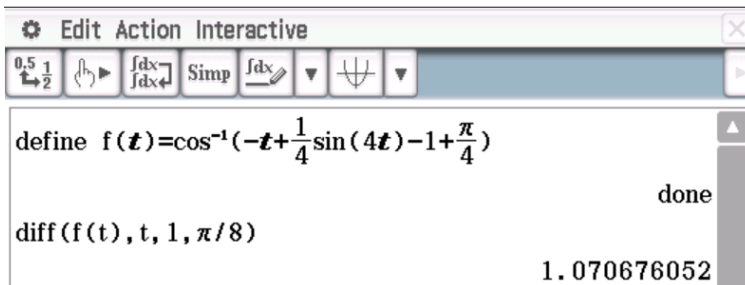
2A

b.  $x(t) = \cos^{-1}\left(-t + \frac{1}{4}\sin(4t) - 1 + \frac{\pi}{4}\right)$

$x'\left(\frac{\pi}{8}\right) = \cos^{-1}\left(-\frac{\pi}{8} + \frac{1}{4}\sin\left(\frac{\pi}{2}\right) - 1 + \frac{\pi}{4}\right)$

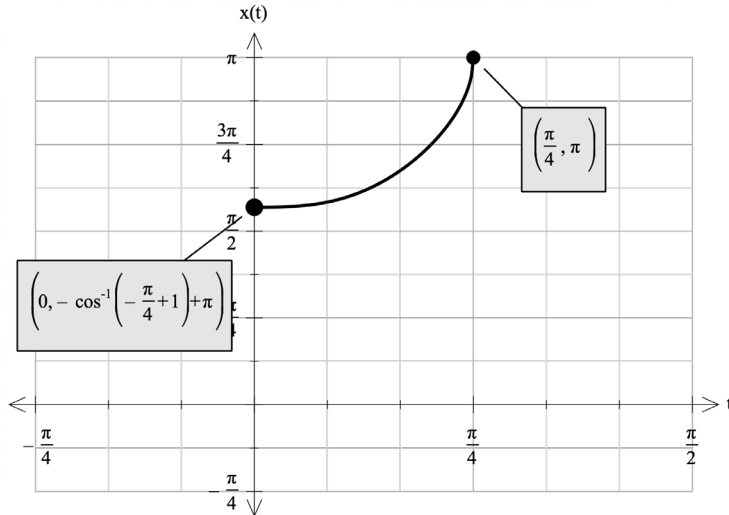
Speed =  $\left|x'\left(\frac{\pi}{8}\right)\right| = 1.07 \text{ ms}^{-1}$  to two decimal places

1A





c.



3A  
1 shape, 2 endpoints

**Question 3 (11 marks)**

a. Use appropriate trigonometric formulas to show that  $\text{cis}\left(\frac{5\pi}{12}\right)$  can be expressed in the form  $A + Bi$ ,

where  $A, B \in \mathbb{R}$  as  $\text{cis}\left(\frac{5\pi}{12}\right) = \frac{\sqrt{6} - \sqrt{2}}{4} + \frac{\sqrt{6} + \sqrt{2}}{4}i$ .

$$\begin{aligned} \text{cis}\left(\frac{5\pi}{12}\right) &= \text{cis}\left(\frac{2\pi}{12} + \frac{3\pi}{12}\right) = \text{cis}\left(\frac{\pi}{6} + \frac{\pi}{4}\right) \\ \text{cis}\left(\frac{\pi}{6} + \frac{\pi}{4}\right) &= \cos\left(\frac{\pi}{6} + \frac{\pi}{4}\right) + i \sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right) \end{aligned}$$

Taking Real components

$$\cos\left(\frac{\pi}{6} + \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}\right)$$

$$\therefore \cos\left(\frac{5\pi}{12}\right) = \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} - \frac{1}{2} \times \frac{\sqrt{2}}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

and taking Imaginary components

$$i \sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right) = i \sin\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{4}\right) + i \cos\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}\right)$$

1M

$$\therefore i \sin\left(\frac{5\pi}{12}\right) = i \frac{1}{2} \times \frac{\sqrt{2}}{2} + i \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} = \frac{\sqrt{2} + \sqrt{6}}{4}i$$

Giving

$$\text{cis}\left(\frac{5\pi}{12}\right) = \frac{\sqrt{6} - \sqrt{2}}{4} + \frac{\sqrt{6} + \sqrt{2}}{4}i \text{ where } A = \frac{\sqrt{6} - \sqrt{2}}{4}, B = \frac{\sqrt{6} + \sqrt{2}}{4}$$

1M

b. Express  $\left\{ z : \left| z - 3 \operatorname{cis} \left( \frac{5\pi}{12} \right) \right| = \left| z + 3 \operatorname{cis} \left( \frac{5\pi}{12} \right) \right|, z \in C \right\}$  in the form  $y = ax + b$ , where  $a, b \in R$ .

$$\left\{ z : \left| z - 3 \operatorname{cis} \left( \frac{5\pi}{12} \right) \right| = \left| z + 3 \operatorname{cis} \left( \frac{5\pi}{12} \right) \right|, z \in C \right\}$$

From **part a**).

$$\text{Let } 3 \operatorname{cis} \left( \frac{5\pi}{12} \right) = \operatorname{cis} \left( \frac{5\pi}{12} \right) = \frac{3\sqrt{6} - 3\sqrt{2}}{4} + i \frac{3\sqrt{6} + 3\sqrt{2}}{4} \quad \mathbf{1M}$$

$$\text{So } \left\{ z : \left| z - 3 \operatorname{cis} \left( \frac{5\pi}{12} \right) \right| = \left| z + 3 \operatorname{cis} \left( \frac{5\pi}{12} \right) \right|, z \in C \right\}$$

$$\left| z - \left( \frac{3\sqrt{6} - 3\sqrt{2}}{4} + i \frac{3\sqrt{6} + 3\sqrt{2}}{4} \right) \right| = \left| z + \frac{3\sqrt{6} - 3\sqrt{2}}{4} + i \frac{3\sqrt{6} + 3\sqrt{2}}{4} \right|$$

Let  $z = x + iy$

$$\left| x + iy - \left( \frac{3\sqrt{6} - 3\sqrt{2}}{4} + i \frac{3\sqrt{6} + 3\sqrt{2}}{4} \right) \right| = \left| x + iy + \frac{3\sqrt{6} - 3\sqrt{2}}{4} + i \frac{3\sqrt{6} + 3\sqrt{2}}{4} \right|$$

$$\sqrt{\left( x - \frac{3\sqrt{6} - 3\sqrt{2}}{4} \right)^2 + \left( y - \frac{3\sqrt{6} + 3\sqrt{2}}{4} \right)^2} = \sqrt{\left( x + \frac{3\sqrt{6} - 3\sqrt{2}}{4} \right)^2 + \left( y + \frac{3\sqrt{6} + 3\sqrt{2}}{4} \right)^2}$$

$$\left( x - \frac{3\sqrt{6} - 3\sqrt{2}}{4} \right)^2 + \left( y - \frac{3\sqrt{6} + 3\sqrt{2}}{4} \right)^2 = \left( x + \frac{3\sqrt{6} - 3\sqrt{2}}{4} \right)^2 + \left( y + \frac{3\sqrt{6} + 3\sqrt{2}}{4} \right)^2 \quad \mathbf{1M}$$

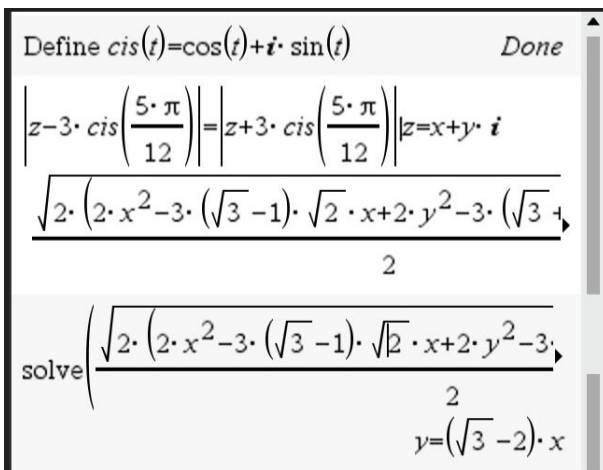
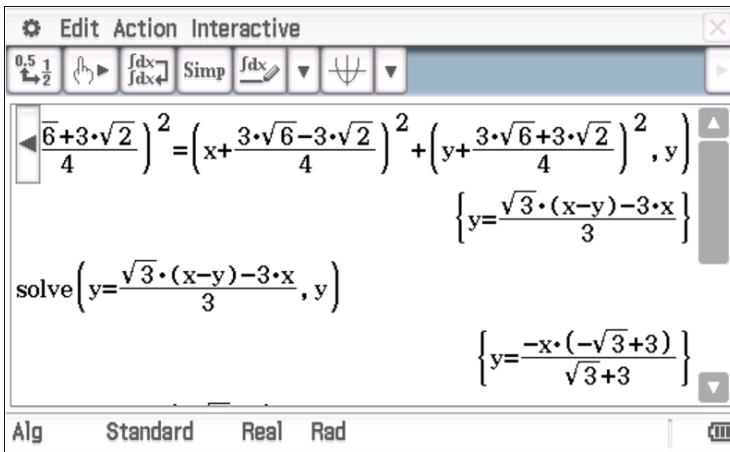
Giving

$$y = -x \left( \frac{3 - \sqrt{3}}{3 + \sqrt{3}} \right)$$

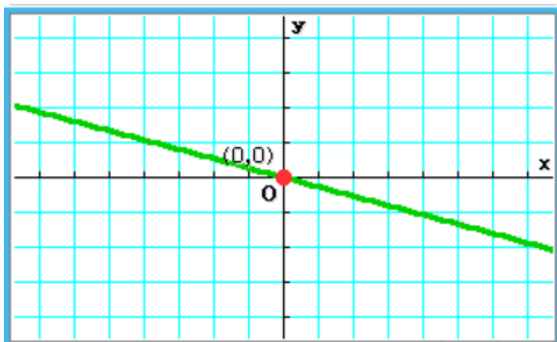
In rationalised form

$$y = (\sqrt{3} - 2)x \quad \mathbf{1A}$$

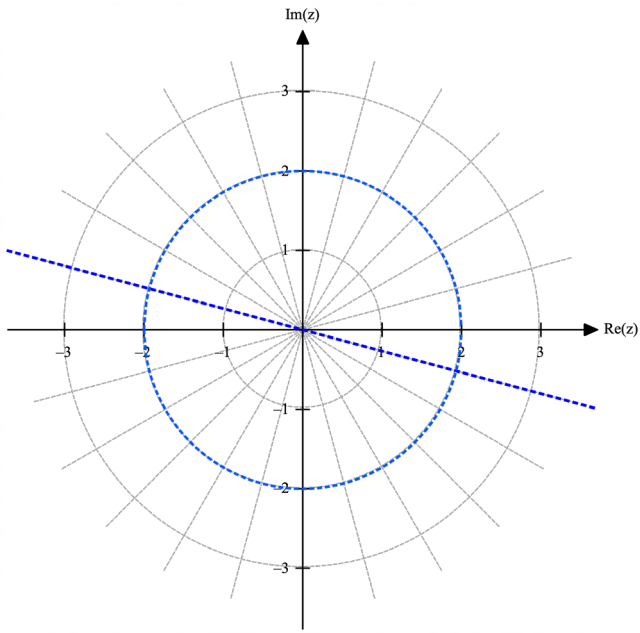
Edit Action Interactive  
 $\text{solve} \left( \left( x - \frac{3\sqrt{6} - 3\sqrt{2}}{4} \right)^2 + \left( y - \frac{3\sqrt{6} + 3\sqrt{2}}{4} \right)^2 = \left( x + \frac{3\sqrt{6} - 3\sqrt{2}}{4} \right)^2 + \left( y + \frac{3\sqrt{6} + 3\sqrt{2}}{4} \right)^2, y \right)$   
 $\left\{ y = \frac{\sqrt{3} \cdot (x - y) - 3 \cdot x}{3} \right\}$   
 $\text{solve} \left( y = \frac{\sqrt{3} \cdot (x - y) - 3 \cdot x}{3}, y \right)$   
 $\left\{ y = \frac{-x \cdot (-\sqrt{3} + 3)}{\sqrt{3} + 3} \right\}$



- c. Sketch the graph of  $\left\{ z : \left| z - 3 \operatorname{cis}\left(\frac{5\pi}{12}\right) \right| = \left| z + 3 \operatorname{cis}\left(\frac{5\pi}{12}\right) \right|, z \in C \right\}$  in the form  $y = ax + b$ , where  $a, b \in R$ , 1 mark



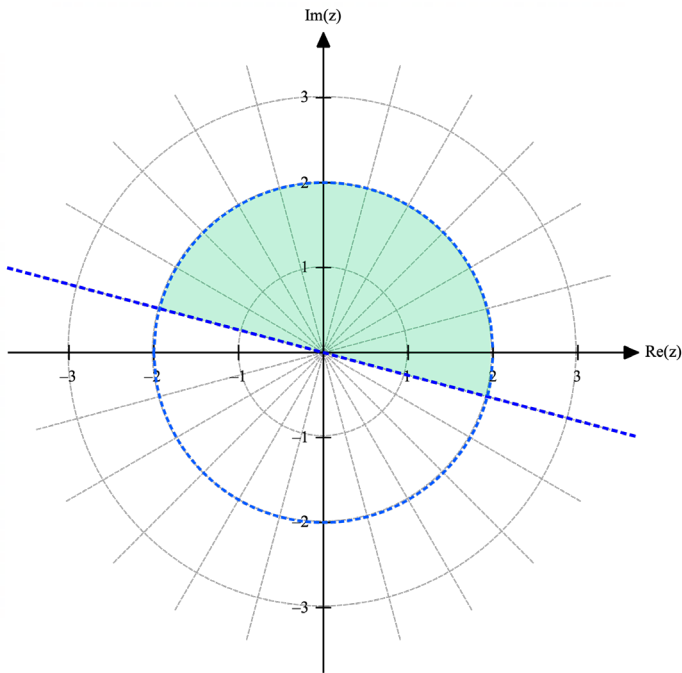
- d. On the Argand diagram below sketch and label  $A = \left\{ z : z \bar{z} = 4, z \in C \right\}$  and  $B = \left\{ z : \left| z - 3 \operatorname{cis}\left(\frac{5\pi}{12}\right) \right| = \left| z + 3 \operatorname{cis}\left(\frac{5\pi}{12}\right) \right|, z \in C \right\}$ .



2A

e. i. Shade of region defined by  $\left\{ z : z \bar{z} \leq 4, z \in C \right\} \cap \left\{ z : \left| z - 3 \operatorname{cis} \left( \frac{5\pi}{12} \right) \right| \leq \left| z + 3 \operatorname{cis} \left( \frac{5\pi}{12} \right) \right|, z \in C \right\}$

1A



ii Intersection points on graph above

$$\left\{ z : z \bar{z} \leq 4, z \in C \right\} \cap \left\{ z : \left| z - 3 \operatorname{cis} \left( \frac{5\pi}{12} \right) \right| \leq \left| z + 3 \operatorname{cis} \left( \frac{5\pi}{12} \right) \right|, z \in C \right\}$$

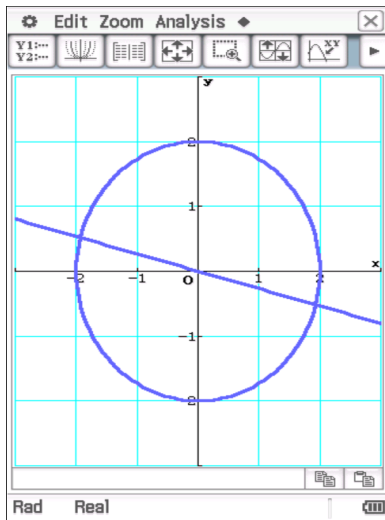
Intersection between line  $y = (\sqrt{3} - 2)x$  and circle  $x^2 + y^2 = 4$

$$x = \frac{2}{\sqrt{6}-\sqrt{2}}, y = \frac{-(\sqrt{6}-\sqrt{2})}{2}$$

$$x = \frac{-2}{\sqrt{6}-\sqrt{2}}, y = \frac{\sqrt{6}-\sqrt{2}}{2}$$

1A

$z \cdot \text{conj}(z) = 4 \implies |z| = 2$   $x^2 + y^2 = 4$   
 $z = x + yi$   
 $z \cdot \text{conj}(z) = (x + yi)(x - yi) = x^2 - (yi)^2 = x^2 + y^2 = 4$   
 $z = \sqrt{2} \cdot (2 \cdot x^2 - 3 \cdot (\sqrt{3} - 1) \cdot \sqrt{2})$   
 $y = (\sqrt{3} - 2) \cdot \sqrt{\sqrt{3} + 2}$  and  $x = \sqrt{\sqrt{3} + 2}$  or  $y = -(\sqrt{3} - 2) \cdot \sqrt{\sqrt{3} + 2}$  and  $x = \sqrt{\sqrt{3} + 2}$  or  $y = -(\sqrt{3} - 2) \cdot \sqrt{\sqrt{3} + 2}$  and  $x = -\sqrt{\sqrt{3} + 2}$   
 $x = \frac{-\sqrt{6}}{2} - \frac{\sqrt{2}}{2}$  and  $y = \frac{(\sqrt{3} - 1) \cdot \sqrt{2}}{2}$  or  $x = \frac{\sqrt{6}}{2}$



1A

$\text{solve}(x^2 + y^2 = 4, y)$   
 $\{y = -\sqrt{-x^2 + 4}, y = \sqrt{-x^2 + 4}\}$   
 $\text{define } f(x) = (\sqrt{3} - 2)x$   
 $\text{done}$   
 $\text{solve}(-\sqrt{-x^2 + 4} = f(x), x)$   
 $\{x = \frac{2}{\sqrt{6} - \sqrt{2}}\}$   
 $\text{solve}(\sqrt{-x^2 + 4} = f(x), x)$   
 $\{x = \frac{-2}{\sqrt{6} - \sqrt{2}}\}$   
 $\text{simplify}(f(\frac{2}{\sqrt{6} - \sqrt{2}}))$   
 $\frac{-(\sqrt{6} - \sqrt{2})}{2}$   
 $\text{simplify}(f(\frac{-2}{\sqrt{6} - \sqrt{2}}))$   
 $\frac{\sqrt{6} - \sqrt{2}}{2}$

f. Find the area of the shaded region in **part e**. Line goes through centre of circle radius of 2.

$$\begin{aligned} \text{Area} &= \frac{1}{2} \pi 2^2 \\ &= 2\pi \end{aligned}$$

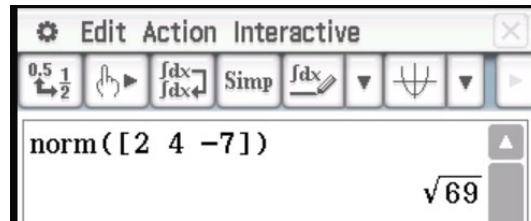
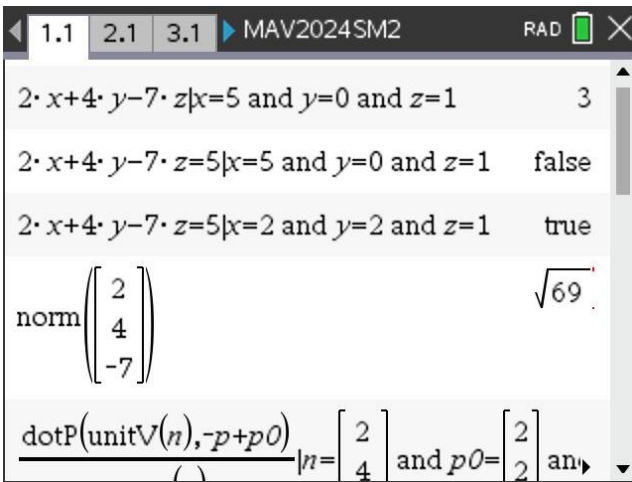
1A

**Question 4**

a. Substituting  $x = 5, y = 0$  and  $z = 1$  into the plane equation gives

$$2x + 4y - 7z = 2 \times 5 + 4 \times 0 - 7 \times 1 = 3 \neq 5 \quad \mathbf{1M}$$

Therefore,  $P(5, 0, 1)$  does not lie on the plane. **1A**



b. The vector perpendicular to the plane is given by the coefficients of  $x, y$  and  $z$  thus it could be  $2\mathbf{i} + 4\mathbf{j} - 7\mathbf{k}$ . **1A**

c. Let  $\mathbf{n} = 2\mathbf{i} + 4\mathbf{j} - 7\mathbf{k}$  where  $\mathbf{n}$  is a normal vector to the plane.

$$|\mathbf{n}| = \sqrt{2^2 + 4^2 + (-7)^2} = \sqrt{69}$$

Find any point  $P_0$  that lies on the plane.

$P_0(2, 2, 1)$  where  $P_0$  is a point on the plane

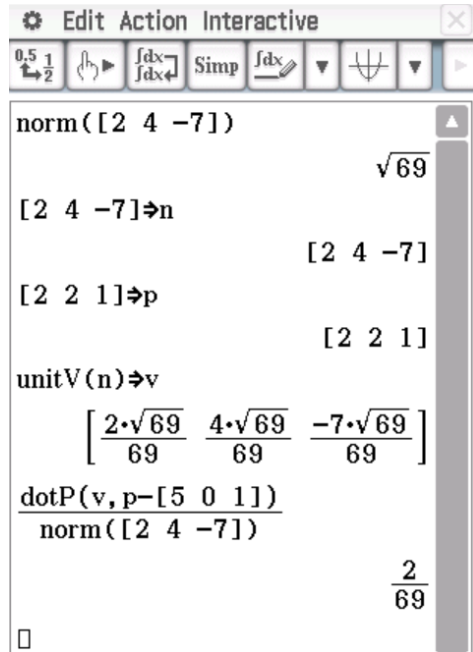
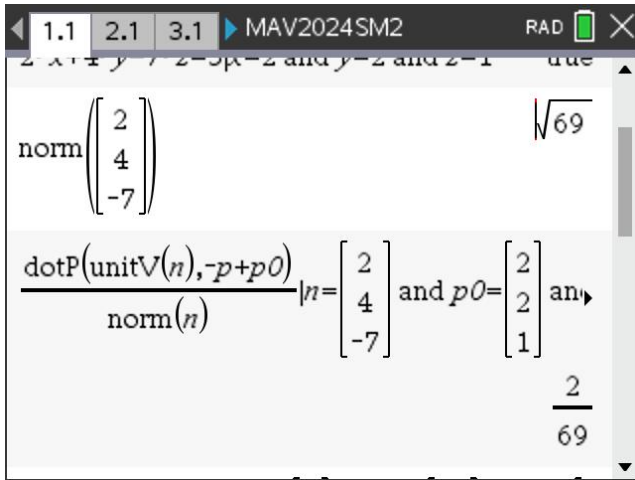
$$\overrightarrow{OP_0} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$$

$$\overrightarrow{PP_0} = \overrightarrow{PO} + \overrightarrow{OP_0} = -(5\mathbf{i} + \mathbf{k}) + (2\mathbf{i} + 2\mathbf{j} + \mathbf{k}) = -3\mathbf{i} + 2\mathbf{j}$$

$$\mathbf{n} \cdot \overrightarrow{PP_0} = (2\mathbf{i} + 4\mathbf{j} - 7\mathbf{k}) \cdot (-3\mathbf{i} + 2\mathbf{j}) = 2 \quad \mathbf{1M}$$

$$D = \frac{\mathbf{n} \cdot \overrightarrow{PP_0}}{|\mathbf{n}|} = \frac{1}{\sqrt{69}} \times 2 = \frac{2}{\sqrt{69}} = \frac{2\sqrt{69}}{69}$$

Therefore, the shortest distance from  $P(5, 0, 1)$  to  $\Pi_1$  is  $\frac{2}{\sqrt{69}}$  (or  $\frac{2\sqrt{69}}{69}$ ). **1A**



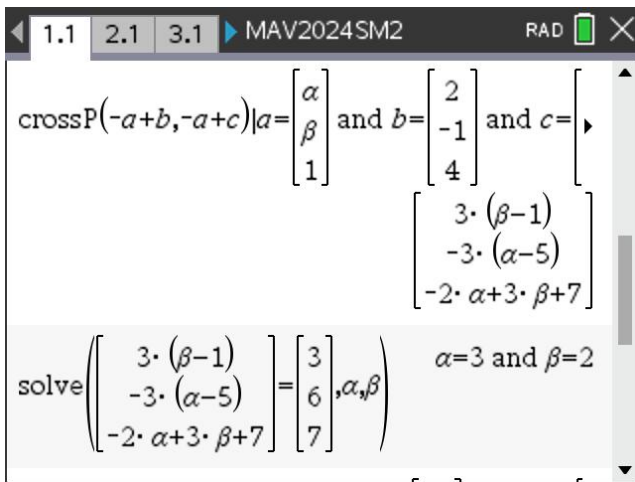
d.  $\overline{AB} = (2\hat{i} - \hat{j} + 4\hat{k}) - (\alpha\hat{i} + \beta\hat{j} + \hat{k}) = (2 - \alpha)\hat{i} + (-1 - \beta)\hat{j} + 3\hat{k}$   
 $\overline{AC} = (5\hat{i} + \hat{j} + \hat{k}) - (\alpha\hat{i} + \beta\hat{j} + \hat{k}) = (5 - \alpha)\hat{i} + (1 - \beta)\hat{j}$       1A

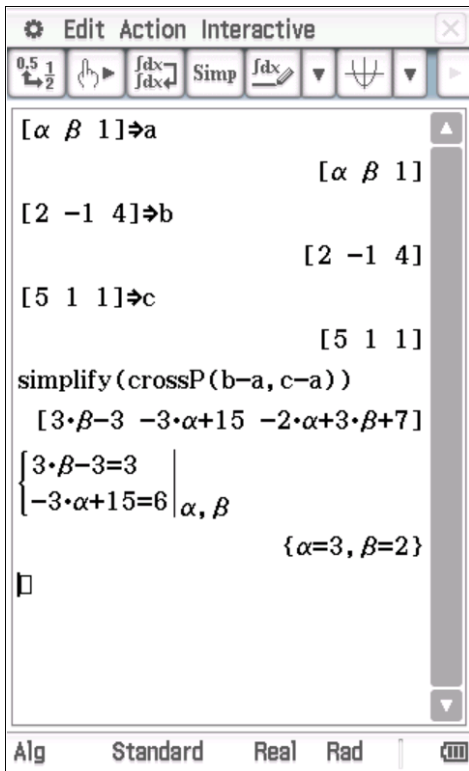
e. Note that  $\hat{r} \cdot (3\hat{i} + 6\hat{j} + 7\hat{k}) = 28 \equiv 3x + 6y + 7z = 28$

A normal vector to the plane is  $\overline{AB} \times \overline{AC} = (3\beta - 3)\hat{i} + (15 - 3\alpha)\hat{j} + (-2\alpha + 3\beta + 7)\hat{k}$ . 1M

Equating  $3\hat{i} + 6\hat{j} + 7\hat{k} = (3\beta - 3)\hat{i} + (15 - 3\alpha)\hat{j} + (-2\alpha + 3\beta + 7)\hat{k}$  will find  $\alpha = 3$  and  $\beta = 2$ .

Therefore, the coordinate of A is (3, 2, 1). 1A



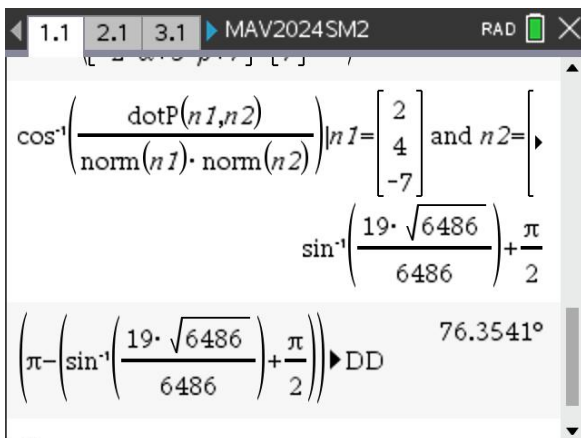


f. A normal to plane  $\Pi_1$  is  $\underline{n}_1 = 2\hat{i} + 4\hat{j} - 7\hat{k}$  and a normal to the plane  $\Pi_2$  is  $\underline{n}_2 = 3\hat{i} + 6\hat{j} + 7\hat{k}$ .  
Let  $\theta$  be the angle between the two planes.

$$\begin{aligned} \underline{n}_1 \cdot \underline{n}_2 &= |\underline{n}_1| |\underline{n}_2| \cos(\theta) \\ -19 &= \sqrt{69} \cdot \sqrt{94} \cdot \cos(\theta) \\ \cos(\theta) &= \frac{-19}{\sqrt{69} \cdot \sqrt{94}} \quad \mathbf{1M} \\ \theta &= 1.809 = 103.646^\circ \end{aligned}$$

The angle between  $\underline{n}_1$  and  $\underline{n}_2$  is  $103.646^\circ = 76.354^\circ = 76.35^\circ$ .

Therefore, the acute angle between the two planes is  $76.35^\circ$ . **1A**



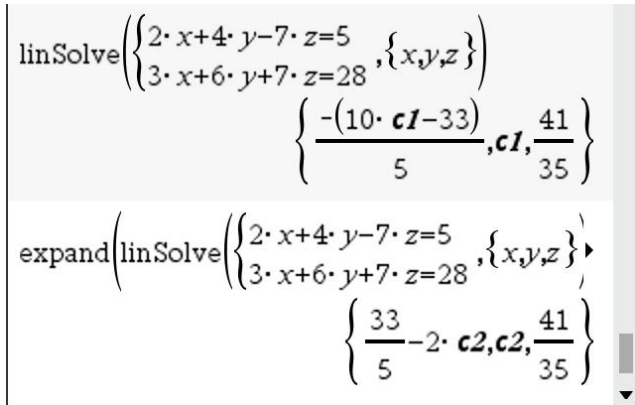
g. Solve  $2x + 4y - 7z = 5$  and  $3x + 6y + 7z = 28$   
would have  $x = -2y + \frac{33}{5}$  (or  $y = -\frac{x}{2} + \frac{33}{10}$ ) and  $z = \frac{41}{35}$ .



Let  $y = t$ . **1M**

We would have  $x = -2t + \frac{33}{5}$ ,  $y = t$  and  $z = \frac{41}{35}$ .

Therefore,  $\underline{\ell} = \left( \frac{33}{5} \underline{i} + 0 \underline{j} + \frac{41}{35} \underline{k} \right) + t(-2 \underline{i} + \underline{j} + 0 \underline{k})$ . **1A**



OR

$$\underline{u} = \underline{n}_1 \times \underline{n}_2 = (2\underline{i} + 4\underline{j} - 7\underline{k}) \times (3\underline{i} + 6\underline{j} + 7\underline{k}) = 70\underline{i} - 35\underline{j}$$
 **1M**

Let  $x = 0$ ,  $2 \times 0 + 4y - 7z = 5$  and  $3 \times 0 + 6y + 7z = 28$ .

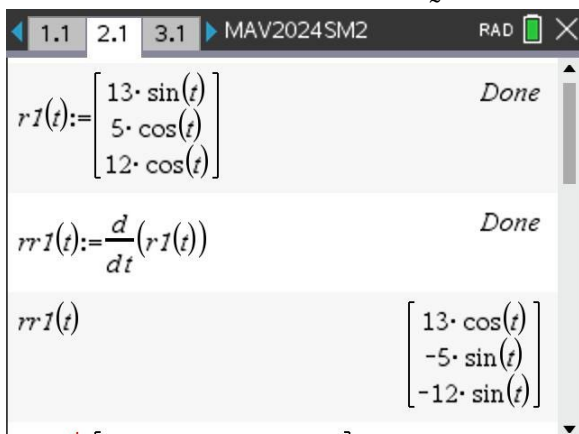
Therefore, the line of intersection is

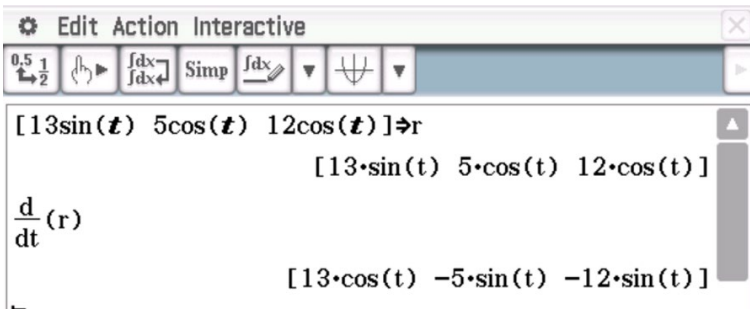
$$\begin{aligned} & \left( 0 \underline{i} + \frac{33}{10} \underline{j} + \frac{41}{35} \underline{k} \right) + t(70 \underline{i} - 35 \underline{j} + 0 \underline{k}) \\ & \equiv \left( 0 \underline{i} + \frac{33}{10} \underline{j} + \frac{41}{35} \underline{k} \right) + \left( \frac{33}{5} \div 70 \right) (70 \underline{i} - 35 \underline{j} + 0 \underline{k}) + \frac{1}{-35} \times t(70 \underline{i} - 35 \underline{j} + 0 \underline{k}) \\ & = \left( 0 \underline{i} + \frac{33}{10} \underline{j} + \frac{41}{35} \underline{k} \right) + \left( \frac{33}{350} \right) (70 \underline{i} - 35 \underline{j} + 0 \underline{k}) + t(-2 \underline{i} + 1 \underline{j} + 0 \underline{k}) \\ & = \left( \frac{33}{5} \underline{i} + 0 \underline{j} + \frac{41}{35} \underline{k} \right) + t(-2 \underline{i} + 1 \underline{j} + 0 \underline{k}) \end{aligned}$$

**1A**

**Question 5**

a.  $\dot{\underline{r}}(t) = 13 \cos(t) \underline{i} - 5 \sin(t) \underline{j} - 12 \sin(t) \underline{k}$





b.

$$\begin{aligned}
 |\dot{r}_1(t)| &= |13 \cos(t)\underline{i} - 5 \sin(t)\underline{j} - 12 \sin(t)\underline{k}| \\
 &= \sqrt{(13 \cos(t))^2 + (-5 \sin(t))^2 + (-12 \sin(t))^2} \\
 &= \sqrt{13^2 \cos^2(t) + 25 \sin^2(t) + 144 \sin^2(t)} \\
 &= \sqrt{13^2 \cos^2(t) + 169 \sin^2(t)} \quad \mathbf{1A} \\
 &= \sqrt{13^2 (\cos^2(t) + \sin^2(t))} \\
 &= \sqrt{13^2 \cdot 1} \\
 &= 13
 \end{aligned}$$

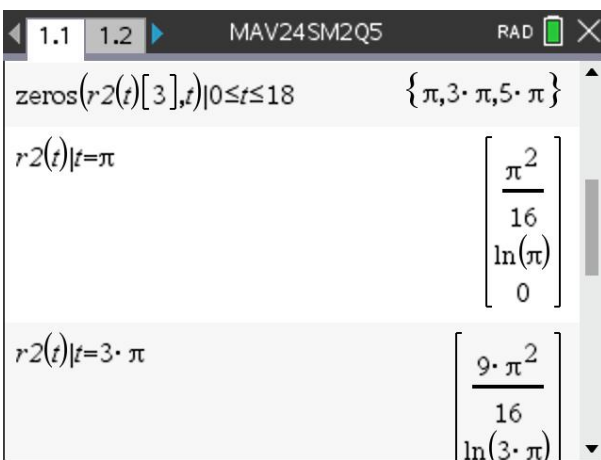
As the speed of this dragon is always 13 and is not dependent on time, thus the speed of this dragon is constant.

**1A**

c. The dragon passes through the xy plane when  $z = 0$ .

When  $\cos\left(\frac{t}{2}\right), t = \pi$ . **1M**

$$r_2(\pi) = \left(\frac{\pi^2}{16}, \log_e \pi, 0\right) \quad \mathbf{1A}$$

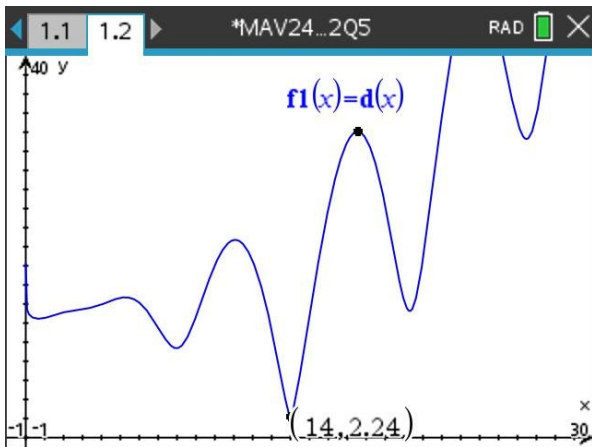
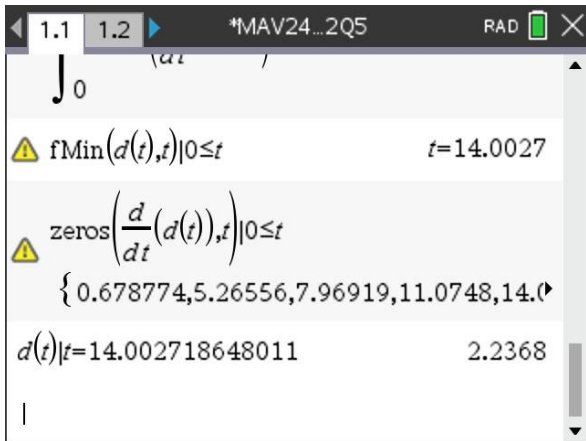


d.  $D(t) = |r_2(t) - r_1(t)|$  **1M**

$$= \sqrt{\begin{matrix} 256 \cos^2\left(\frac{t}{2}\right) - 6144 \cos(t) \cos\left(\frac{t}{2}\right) \\ -1280 \left(\log_e(t) - \log_e\left(\frac{1}{t}\right)\right) \cos(t) \\ -416t^2 \sin(t) - 256 \log_e\left(\frac{1}{t}\right) \log_e(t) \\ +t^4 + 43264 \end{matrix}} \quad \div 16 \quad (\text{students are not expected to copy this})$$

$D(t)$  is minimum when  $t = 14.002718648011 \approx 14.003$ . 1A

Thus the minimum distance is  $D(14.0027\dots) = 2.2368024196637 \approx 2.237$  1A



e.  $\dot{\mathbf{r}}_2(t) = (-2 \sin(t) \cos(t) - 6 \sin(t))\mathbf{i} + (-2 \cos^2(t) + 6 \cos(t) + 1)\mathbf{j} + \left(\frac{1}{t^2 + 1}\right)\mathbf{k}$  1M

The distance travelled by the second dragon is given by

$$\int_1^3 |\dot{\mathbf{r}}_2(t)| = \int_1^3 \sqrt{\left(\frac{t}{8}\right)^2 + \left(\frac{1}{t}\right)^2 + \left(\frac{-\sin \frac{t}{2}}{2}\right)^2} dt \quad 1A$$

$$= 1.50019 = 1.500$$

$$\int_1^3 \text{norm}\left(\frac{d}{dt}(r_2(t))\right) dt = 1.50019$$

**Question 6**

a. Let the sales figure be  $K$  on that particular day.

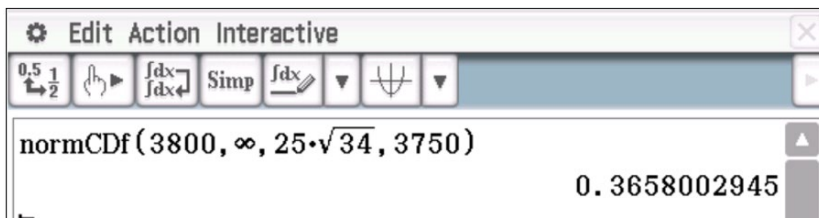
$$E(K) = 3000 + (-10) \times 25 + 5 \times 200 = 3000 - 250 + 1000 = 3750 \quad \mathbf{1A}$$

$$\text{Var}(K) = 25^2 \times 5^2 + 5^2 \times 15^2 = 21250$$

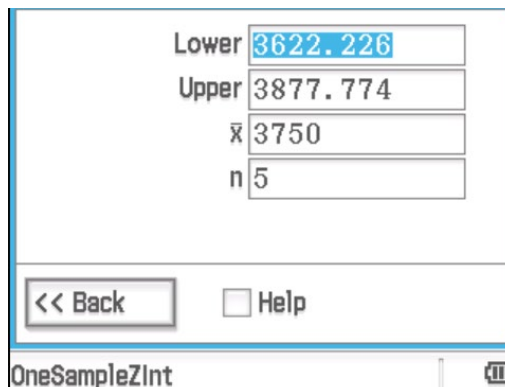
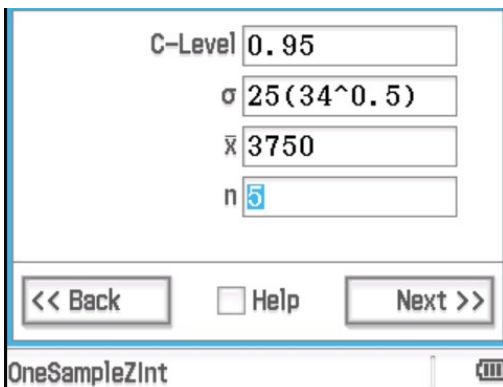
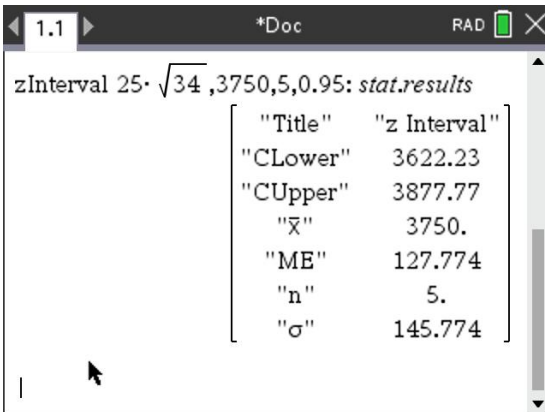
$$\text{SD}(K) = \sqrt{\text{Var}(K)} = \sqrt{21250} = 25\sqrt{34} \quad \mathbf{1A}$$

Thus,  $K \sim N(3750, 21250)$

b.  $\Pr(K > 3800) = 0.36580 = 0.3658 \quad \mathbf{1A}$

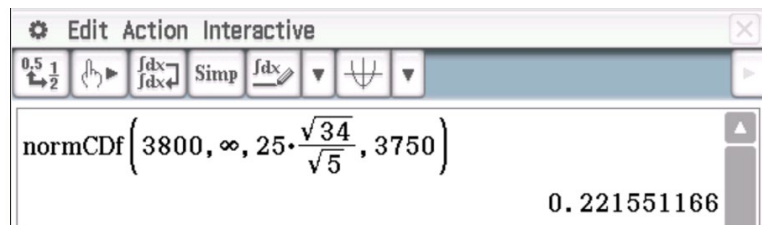
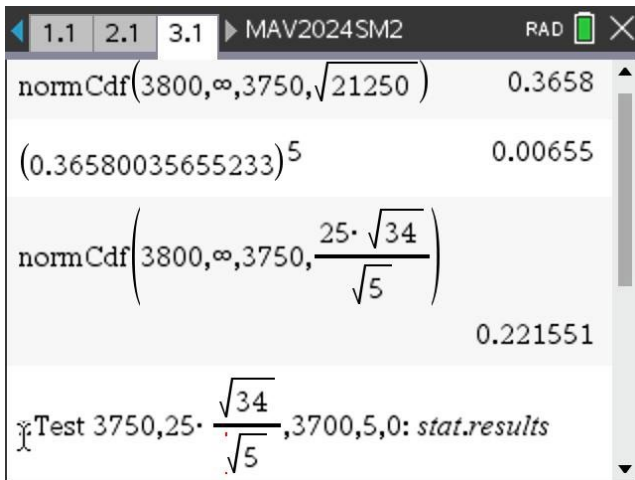


c.  $(3622.226, 3877.774) \quad \mathbf{1A}$



d.  $\bar{K} \sim N\left(3750, \left(\frac{25\sqrt{34}}{\sqrt{5}}\right)^2\right)$  **1M**

$\Pr(\text{ave. Saturdays exceed } 3800) = \Pr(\bar{K} > 3800) = 0.2216$  **1A**



e.  
 $H_0 : \mu = 3700$   
 $H_1 : \mu < 3700$  **1A**

f.  $p$ -value  $\Pr(k < 3700 | \mu = 3750) = 0.2215$  **1M**

We thus do not reject the null hypothesis. There could be some truth in owner’s claim. **1A**

**END OF QUESTION AND ANSWER BOOK SOLUTIONS**