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SPECIALIST MATHEMATICS UNITS 3 & 4

TRIAL EXAMINATION 2

2024

Reading Time: 15 minutes
Writing time: 2 hours

Instructions to students

This exam consists of Section A and Section B.
Section A consists of 20 multiple-choice questions and should be answered on the answer sheet which can be found on page 26 of this exam.
Section B consists of 6 extended-answer questions.
Section A begins on page 2 of this exam and is worth 20 marks.
Section B begins on page 11 of this exam and is worth 60 marks.
There is a total of 80 marks available.
All questions in Section A and B should be answered.
In Section B, where more than one mark is allocated to a question, appropriate working must be shown.
An exact value is required to a question unless otherwise directed.
Unless otherwise stated, diagrams in this exam are not drawn to scale.
Students may bring one bound reference into the exam.
Students may bring into the exam one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory does not need to be cleared. For approved computer-based CAS, full functionality may be used.
A formula sheet can be found at the end of this exam.

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SECTION A – Multiple-choice questions

Question 1

The negation of the statement '$\exists n \in Z$ such that $n^2 + 6n + 5 = 0$' is

A. $\forall n \in Z, n^2 + 6n + 5 = 0$
B. $\exists n \in Z$ s.t. $n^2 + 6n + 5 \neq 0$
C. $\exists n \notin Z$ s.t. $n^2 + 6n + 5 \neq 0$
D. $\forall n \in Z, n^2 + 6n + 5 \neq 0$

Question 2

The statement 'For any $n \in Z$, prove that if n^2 is even, then n is even' is most easily proved using

A. direct proof
B. contrapositive
C. mathematical induction
D. using the converse

Question 3

The following pseudocode uses Euler's method to find the values of x_n and y_n for a given differential equation $\frac{dy}{dx} = f(x, y)$. The pseudocode takes input of a point (x_0, y_0), a step size of *h* and the number of iteration steps to take, n.

```
Define f(x, y) :
        Return x²/y

Define euler(x0,y0,h,n):
        x ← x0
        y ← y0
        for i from 1 to n
                <INSERT LINE1 HERE>
                <INSERT LINE2 HERE>
        end for
        return (x, y)
```

The two lines of code missing above where it says <INSERT LINE HERE> are

A.
$x \leftarrow x + h \times f(x, y)$
$y \leftarrow y + h$

B.
$y \leftarrow y + h$
$x \leftarrow x + h \times f(x, y)$

C.
$y \leftarrow y + h \times f(x, y)$
$x \leftarrow x + h$

D.
$x \leftarrow x + h$
$y \leftarrow y + h \times f(x, y)$

Question 4

The domain and range of the function with the rule $f(x) = b\arccos\left(\frac{x}{2}\right) - \frac{\pi}{4}$ for $b \in R^+$, are respectively

A. $[-4,4]$ and $\left[\frac{-\pi}{4}, \frac{5\pi}{4}\right]$

B. $[-2,2]$ and $\left[\frac{-\pi}{4}, b\pi\right]$

C. $[-2,2]$ and $\left[\frac{-\pi}{4}, b\pi - \frac{\pi}{4}\right]$

D. $[-1,1]$ and $[0,\pi]$

Question 5

The area of the triangle ABC with vertices $A(1,2,-1)$, $B(4,3,2)$ and $C(-3,-1,4)$ is

A. $\frac{5\sqrt{6}}{2}$

B. $\frac{\sqrt{705}}{2}$

C. $\frac{5\sqrt{38}}{2}$

D. $\frac{5\sqrt{2}}{2}$

Question 6

Two lines are defined by

$$\underset{\sim}{r}_1(\lambda) = 2\underset{\sim}{i} + 3\underset{\sim}{j} - \underset{\sim}{k} + \lambda(\underset{\sim}{i} + 3\underset{\sim}{j} + 3\underset{\sim}{k})$$
$$\underset{\sim}{r}_2(\mu) = 5\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k} + \mu(2\underset{\sim}{i} + 4\underset{\sim}{j} + a\underset{\sim}{k})$$

where $\lambda, \mu, a \in R$.

If the two lines are skew, then

A. $a \in R \setminus \left\{\frac{21}{5}\right\}$

B. $a = \frac{23}{5}$

C. $a = \frac{21}{5}$

D. $a \in R \setminus \left\{\frac{23}{5}\right\}$

Question 7

The vectors $\underset{\sim}{a} = 4\underset{\sim}{i} + 3\underset{\sim}{j} - 4\underset{\sim}{k}$, $\underset{\sim}{b} = 3\underset{\sim}{i} + \lambda\underset{\sim}{j} + 2\underset{\sim}{k}$ and $\underset{\sim}{c} = -3\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$ are linearly dependent when the value of λ is

A. $\lambda = -5$
B. $\lambda = -4$
C. $\lambda = 4$
D. $\lambda = 5$

Question 8

Which of the following is a vector that has a length of two units and is perpendicular to $4\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$?

A. $\frac{2}{\sqrt{41}}(-2\underset{\sim}{i} + \underset{\sim}{j} - 6\underset{\sim}{k})$

B. $\frac{2}{\sqrt{41}}(4\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k})$

C. $\frac{2}{41}(2\underset{\sim}{i} - \underset{\sim}{j} + 6\underset{\sim}{k})$

D. $\frac{2}{\sqrt{41}}(2\underset{\sim}{i} + \underset{\sim}{j} - 6\underset{\sim}{k})$

Question 9

Given that $i^m = x$, then i^{2m-4} is equal to

A. $\dfrac{x^2}{i}$

B. ix^2

C. x^2

D. $x^2 + i$

Question 10

If $z = -1 - i$, then z^{2024} is

A. real and positive
B. equal to a positive real multiple of i
C. equal to a negative real multiple of i
D. real and negative

Question 11

The circle $|z - 1 - 2i| = 2$ is intersected exactly twice by the line given by

A. $|z - 3 - 2i| = |z - 5|$

B. $|z - 3 - 2i| = |z - 1 - 4i|$

C. $\text{Re}(z) = 3$

D. $\text{Im}(z) = 0$

Question 12

The position vector of a particle at time $t \geq 0$ is given by $\underset{\sim}{r}(t) = 3\sin(2t)\underset{\sim}{i} + 4\cos(4t)\underset{\sim}{j}$.
The path of the particle has the Cartesian equation

A. $\dfrac{x^2}{9} + \dfrac{(y+4)^2}{64} = 1$

B. $\dfrac{x^2}{3} + \dfrac{(y+4)^2}{4} = 1$

C. $\dfrac{x^2}{9} + \dfrac{y^2}{16} = 1$

D. $\dfrac{x^2}{9} + \dfrac{y+4}{8} = 1$

Question 13

The acceleration of a particle at time t seconds, $t \geq 0$, is given by $\underset{\sim}{a}(t) = 2t\underset{\sim}{i} + 3t^2\underset{\sim}{j} - 4\sqrt{t}\underset{\sim}{k}$. The particle's initial velocity is $2\underset{\sim}{i} + \underset{\sim}{j}$ and its initial position is at the origin. The direction of motion of the particle when $t = 1$ is

A. $3\underset{\sim}{i} + 2\underset{\sim}{j} - \frac{8}{3}\underset{\sim}{k}$

B. $\frac{7}{3}\underset{\sim}{i} + \frac{5}{4}\underset{\sim}{j} - \frac{16}{15}\underset{\sim}{k}$

C. $2\underset{\sim}{i} + \underset{\sim}{j}$

D. $2\underset{\sim}{i} + 4\underset{\sim}{j} - 4\underset{\sim}{k}$

Question 14

The first two points of intersection of the graphs of $y = \sin(x)$ and $y = \cos(2x)$ for $x > 0$ occur when $x = a$ and $x = b$, $a, b \in R^+$. The area between the two curves for $x \in [a, b]$ is

A. $\frac{3\sqrt{3}}{4} - 1$

B. $\frac{\sqrt{3}}{2}$

C. $\frac{3\sqrt{3}}{2}$

D. $\frac{3\sqrt{3}}{4} + 1$

Question 15

Consider the differential equation $\frac{dy}{dx} = 3x^2 y$, with $y(0) = k$, $k \in R$. Using Euler's method with a step size of $h = \frac{1}{3}$, the value of $y_3 = 1$. What is the value of k?

A. $\frac{9}{17}$

B. $\frac{81}{157}$

C. $\frac{10}{9}$

D. $\frac{81}{130}$

Question 16

A particle moves along a horizontal line so that its position, x cm, relative to a point O is given by $x(t) = 3t^3 - 6t + 11$, where t is the time, in seconds, after the motion starts.

The acceleration of the particle, in cm/s^2, when the particle has a speed of 4 cm/s, is

A. $6\sqrt{2}$ only

B. $6\sqrt{10}$ only

C. $6\sqrt{2}$ or $6\sqrt{10}$

D. $\pm\dfrac{\sqrt{10}}{3}$

Question 17

Using a suitable substitution, $\displaystyle\int_0^{\frac{\sqrt{2}}{2}} \frac{\log_e(\arccos(x))}{\sqrt{4-4x^2}}\, dx$ can be expressed as

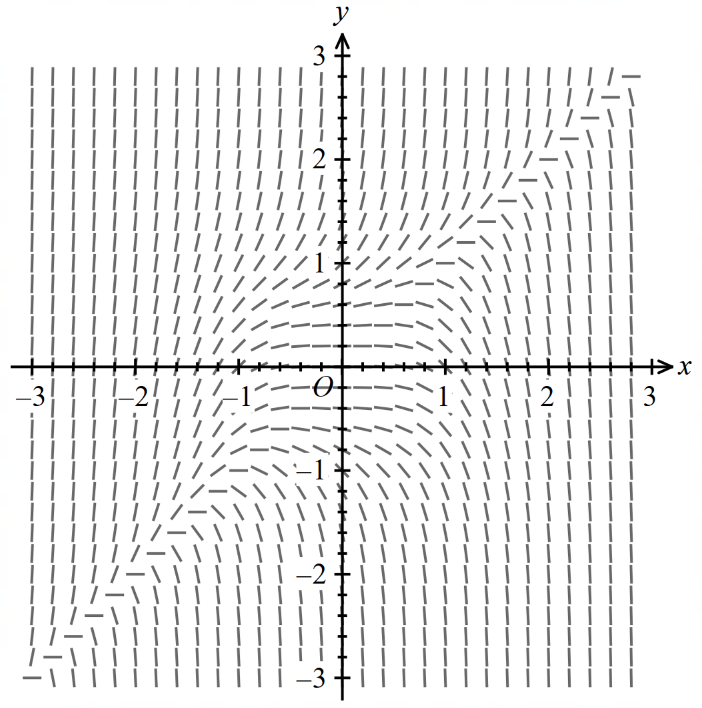
A. $\displaystyle\int_0^{\frac{\pi}{4}} -\log_e(u)\, du$

B. $\displaystyle-\frac{1}{2}\int_0^{\frac{\sqrt{2}}{2}} \log_e(u)\, du$

C. $\displaystyle\frac{1}{2}\int_{\frac{\pi}{2}}^{\frac{\pi}{4}} \log_e(u)\, du$

D. $\displaystyle\frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \log_e(u)\, du$

Question 18



The differential equation that has the diagram above as its directional field is

A. $\dfrac{dy}{dx} = \dfrac{x}{y}$

B. $\dfrac{dy}{dx} = \dfrac{y}{x}$

C. $\dfrac{dy}{dx} = x^3 - y^3$

D. $\dfrac{dy}{dx} = y^3 - x^3$

Question 19

A factory contains four identical machines used in the manufacturing of sporting equipment. The lifetimes of each of these machines are independent and are each normally distributed with a mean of 1100 hours and a standard deviation of 200 hours. The probability that the total lifetime of the four machines is more than 5000 hours is closest to

A. 0.0668
B. 0.2266
C. 0.4256
D. 0.7734

Question 20

Scientists use a confidence interval to estimate the mean height of a population of emus. They take a random sample of emus and calculate the mean height for the sample. This is then used to calculate the confidence interval.

If the scientists are interested in decreasing the width of this confidence interval by 80%, then the sample size should be multiplied by a factor of

A. 5
B. 20
C. 25
D. 80

SECTION B

Question 1 (9 marks)

Consider the function $f : D \to R, f(x) = 20\log_e\left(\frac{x-5}{10}\right)$.

a. Determine the maximal domain D and the range of f. 1 mark

b. The region enclosed by $f(x)$ and the y-axis over the interval $0 \le y \le 30$ is rotated about the y-axis to form a solid of revolution. If x and y are measured in centimetres, state the volume of this solid formed, in cm^3, correct to three decimal places. 3 marks

The solid formed is initially empty and is filled with water to a depth of h centimetres.

c. Find, in cm^3, the volume of water at this depth in terms of h. 1 mark

d. The solid is being filled with the water at a rate of 45 cubic centimetres per minute. Find the rate of change of the depth with respect to time when the depth is 15 centimetres, correct to four decimal places. 2 marks

Now consider the function $g(x) = 20\log_e\left(\frac{ax-b}{c}\right)$ over its maximal domain D_2, where $a, b, c \in R$.

e. **i.** Find the value(s) of a, b and c, such that $g''(x) < 0$ for all $x \in D_2$. 1 mark

ii. Find the required conditions involving a, b and c for which the x-intercept of the graph of $g(x)$ is negative. 1 mark

Question 2 (12 marks)

Let $z_1 = \frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2} i$, $z_1 \in C$.

a. Given that $\text{Arg}(z_1) = \alpha$. State the value of α. 1 mark

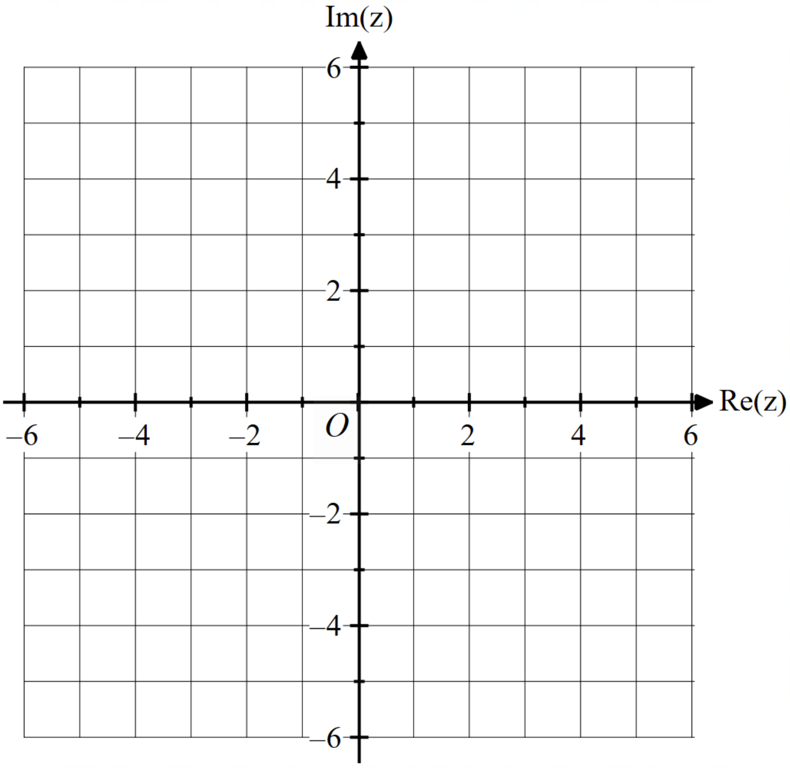
b. **i.** Verify that z_1 is a solution to the equation $z^4 = -81$. 1 mark

ii. Hence or otherwise, solve the equation $z^4 = -81$, $z \in C$, expressing your solutions in polar form. 2 marks

The solutions to the equation $z^4 = -81$ lie on the circle *J* with equation $|z| = 3$.

c. On the Argand diagram below, sketch the circle *J*, as well as the solutions to $z^4 = -81$. Label the solutions with their coordinates.

2 marks



d. On the same set of axes, sketch $\{z : |z - 3 - 3i| = 3, z \in C\}$. 1 mark

e. Find the area of the region defined by

$$\{z : |z - 3 - 3i| \le 3, z \in C\} \cap \{z : |z| \le 3, z \in C\}.$$

2 marks

f. If $|z| \leq 3$ and $|z-3-3i| \leq 3$, find the minimum value of $|z|$. 3 marks

Question 3 (10 marks)

A plane, $\prod_1$, has Cartesian equation $2x-3y+z=15$.

a. Verify that the points $A(0,0,15)$, $B(1,0,13)$ and $C(0,-5,0)$ lie on $\prod_1$. 1 mark

b. Hence, find a vector equation of $\prod_1$ in the form
$\underset{\sim}{r}(s,t)=\underset{\sim}{a}+s\underset{\sim}{u}+t\underset{\sim}{v}$, where $s,t\in R$. 2 marks

The point P has coordinates $(-4,2,5)$.

c. Find the distance of P from $\prod_1$. 3 marks

d. The point P lies on the plane $\prod_2$. If $\prod_2$ is parallel to $\prod_1$, state the Cartesian equation of $\prod_2$. 2 marks

e. The line L passes through the point $W(2, 4, -6)$ and the point P. Determine the point of intersection of L and $\prod_1$. 2 marks

Question 4 (10 marks)

A packet of sausages is taken out of a freezer to defrost. It is left in a room which has a constant temperature of $25°C$.

The temperature $T°C$ of the packet of sausages t minutes after it is removed from the freezer is given by the differential equation

$$\frac{dT}{dt} = -k(T-25)$$

where $k \in R^+$ is a constant.

a. Show that $T = 25 + Ae^{-kt}$, where A is a constant. 3 marks

The packet of sausages came out of the freezer with a temperature of $-10°C$.

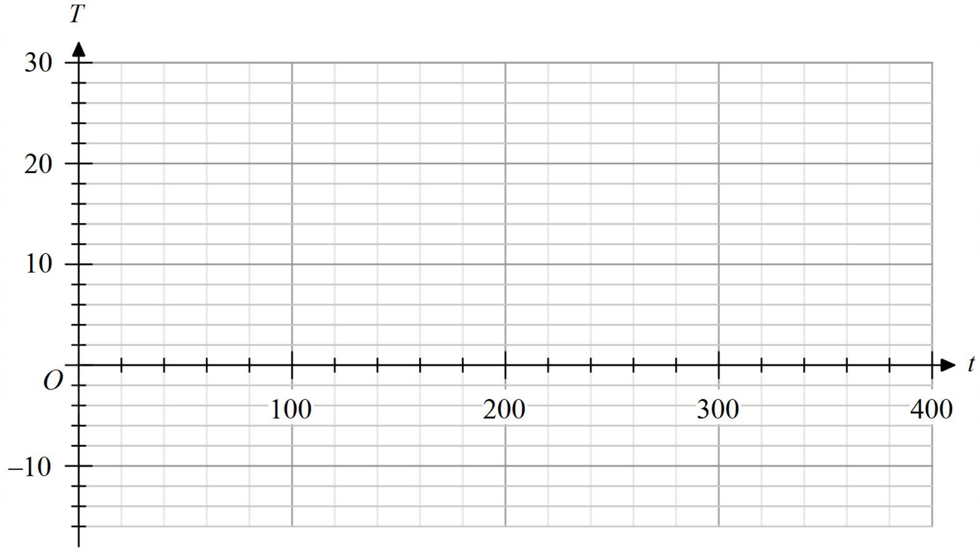
b. Show that $A = -35$. 1 mark

After 30 minutes, the temperature of the packet of sausages is $-2°\text{C}$.

c. Find the rule for the temperature, T, of the packet of sausages at any time t.

Give your answer in the form $T = c - d\left(\frac{f}{d}\right)^{\frac{t}{30}}$, where c, d and f are positive integers. 3 marks

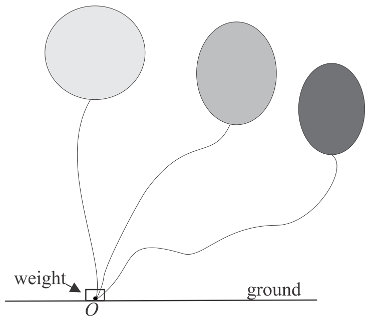
d. On the set of axes below, sketch the graph of T against t for $t \geq 0$.
Label the T-intercept with its coordinates and the asymptote with its equation. 2 marks



e. The sausages are best cooked when they reach $15°\text{C}$. How long after being removed from the freezer should the sausages be left to sit in the room before being cooked? Give your answer in minutes correct to one decimal place. 1 mark

Question 5 (9 marks)

At a child's birthday party, a set of three foil helium balloons are tied to a weight as shown.



Balloon 1 suddenly pops, and a tiny piece of foil from it travels through the air.

The path of this piece of foil from balloon 1, relative to the origin O where the weight rests on the ground, t seconds after the pop, can be modelled by the vector equation

$$\underset{\sim}{b}_1(t) = \cos(5t)\underset{\sim}{i} + \sin(5t)\underset{\sim}{j} + (3t+1)\underset{\sim}{k} \quad \text{for } t \in [0,5]$$

where $\underset{\sim}{i}$ is a unit vector in an easterly direction, $\underset{\sim}{j}$ is a unit vector in a northerly direction and $\underset{\sim}{k}$ is a unit vector vertically up. Displacement components are measured in metres.

a. **i.** State the initial coordinates of balloon 1 and hence the initial height of the balloon. 1 mark

___

___

___

ii. Find the speed of the piece of foil from balloon 1, in m/s, at time t. 2 marks

___

___

___

___

Balloon 2 is untied and begins floating up slowly into the air. This balloon has a small red dot on it and the path of this dot can be modelled by the position vector

$$\underset{\sim}{b}_2(t) = 0.1t\underset{\sim}{i} + 0.5t\underset{\sim}{j} + (1.2t + 1.5)\underset{\sim}{k} \qquad t \geq 0$$

where t is the time in seconds after it is untied and the position is again relative to the origin O on the ground where the weight rests.

b. How far does balloon 2 travel in its first four seconds after being untied?

Use the formula $d = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt$ for an arc length in three dimensions and give your answer in metres correct to two decimal places. 1 mark

A child at the birthday party has a small toy bow and arrow. Three seconds after balloon 2 is untied, the child fires their arrow. The tip of the arrow follows a path given by the position vector

$$\underset{\sim}{a}(t) = (t^2 - 0.9t - 5.7)\underset{\sim}{i} + (9 - 2t)\underset{\sim}{j} + (ct^2 + 2)\underset{\sim}{k} \quad , c \in R$$

where t is the time in seconds after the arrow is fired and the position is again relative to the origin on the ground.

c. Determine the value of c, so that the tip of the arrow hits the red dot on balloon 2. 2 marks

Balloon 3 is spherical in shape with an initial radius of 20 centimetres. It slowly deflates over time, such that its surface area is decreasing at a rate of 35 cm^2/hour.

d. At what rate is the radius of the balloon decreasing with respect to time when the radius is 12 centimetres? Give your answer in units of cm/hour. 2 marks

e. How long does it take, in hours, for the balloon to totally deflate (that is, to have a radius of zero)? 1 mark

Question 6 (10 marks)

The number of visitors to a city zoo each day is normally distributed with a mean of 2500 and a standard deviation of 100.

a. Determine, correct to four decimal places

i. the probability that on any random day, the zoo has more than 2650 visitors. 1 mark

ii. the probability that the average attendance to the zoo over a five day period is greater than 2650. 1 mark

The zoo implements an online ticketing system, and tests whether this **changes** the average number of visitors each day.

b. State the null and alternative hypotheses for this test. 1 mark

To test their hypothesis, management take a sample of four random days and find the average attendance to be 2600.

c. Determine the *p*-value for this test and hence, state whether management should reject the null hypothesis. Test at the 5% level of significance. 2 marks

d. Another sample of size n is collected. The sample mean is found to be 2550. What is the minimum value of n for which H_0 would be rejected? 2 marks

e. If the zoo is testing at the 5% significance level, define a Type I error and state the probability of this error occurring. 1 mark

f. Suppose that the true average attendance is 2600. If the zoo is using a sample of five days and testing at the 5% significance level, find the probability that a Type II error is made. Give your answer correct to **five** decimal places. 2 marks

SPECIALIST MATHS UNITS 3&4

TRIAL EXAMINATION 2

MULTIPLE-CHOICE ANSWER SHEET

STUDENT NAME ....

INSTRUCTIONS

Fill in the letter that corresponds to your choice. Example: A ■ C D

The answer selected is B. Only one answer should be selected.

1.	A	B	C	D	11.	A	B	C	D
2.	A	B	C	D	12.	A	B	C	D
3.	A	B	C	D	13.	A	B	C	D
4.	A	B	C	D	14.	A	B	C	D
5.	A	B	C	D	15.	A	B	C	D
6.	A	B	C	D	16.	A	B	C	D
7.	A	B	C	D	17.	A	B	C	D
8.	A	B	C	D	18.	A	B	C	D
9.	A	B	C	D	19.	A	B	C	D
10.	A	B	C	D	20.	A	B	C	D

Specialist Mathematics formula sheet

Mensuration

area of a circle segment	$\frac{r^2}{2}(\theta - \sin(\theta))$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2} bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$	sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
volume of a pyramid	$\frac{1}{3} Ah$	cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Algebra, number and structure (complex numbers)

$z = x + iy = r(\cos(\theta) + i\sin(\theta)) = r\,\text{cis}(\theta)$	$\lvert z\rvert = \sqrt{x^2 + y^2} = r$	
$-\pi < \text{Arg}(z) \leq \pi$	$z_1 z_2 = r_1 r_2 \,\text{cis}(\theta_1 + \theta_2)$	
$\frac{z_1}{z_2} = \frac{r_1}{r_2}\text{cis}(\theta_1 - \theta_2)$	de Moivre's theorem	$z^n = r^n \text{cis}(n\theta)$

Data analysis, probability and statistics

for independent random variables $X_1, X_2 \ldots X_n$	$\text{E}(aX_1 + b) = a\text{E}(X_1) + b$ $\text{E}(a_1 X_1 + a_2 X_2 + \ldots + a_n X_n)$ $= a_1\text{E}(X_1) + a_2\text{E}(X_2) + \ldots + a_n\text{E}(X_n)$	
	$\text{Var}(aX_1 + b) = a^2\text{Var}(X_1)$ $\text{Var}(a_1 X_1 + a_2 X_2 + \ldots + a_n X_n)$ $= a_1^2\text{Var}(X_1) + a_2^2\text{Var}(X_2) + \ldots + a_n^2\text{Var}(X_n)$	
for independent identically distributed variables $X_1, X_2 \ldots X_n$	$\text{E}(X_1 + X_2 + \ldots + X_n) = n\mu$	
	$\text{Var}(X_1 + X_2 + \ldots + X_n) = n\sigma^2$	
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}},\ \bar{x} + z\frac{s}{\sqrt{n}}\right)$	
distribution of sample mean $\bar{X}$	mean	$\text{E}(\bar{X}) = \mu$
	variance	$\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$	$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$	$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$	$\int \frac{1}{x}\,dx = \log_e\lvert x\rvert + c$
$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$	$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$	$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$	$\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$
$\frac{d}{dx}\left(\cot(ax)\right) = -a\operatorname{cosec}^2(ax)$	$\int \operatorname{cosec}^2(ax)\,dx = -\frac{1}{a}\cot(ax) + c$
$\frac{d}{dx}\left(\sec(ax)\right) = a\sec(ax)\tan(ax)$	$\int \sec(ax)\tan(ax)\,dx = \frac{1}{a}\sec(ax) + c$
$\frac{d}{dx}(\operatorname{cosec}(ax)) = -a\operatorname{cosec}(ax)\cot(ax)$	$\int \operatorname{cosec}(ax)\cot(ax)\,dx = -\frac{1}{a}\operatorname{cosec}(ax) + c$
$\frac{d}{dx}\left(\sin^{-1}(ax)\right) = \frac{a}{\sqrt{1-(ax)^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$
$\frac{d}{dx}\left(\cos^{-1}(ax)\right) = \frac{-a}{\sqrt{1-(ax)^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$
$\frac{d}{dx}\left(\tan^{-1}(ax)\right) = \frac{a}{1+(ax)^2}$	$\int \frac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\frac{x}{a}\right) + c$
	$\int (ax+b)^n\,dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c,\ n \neq -1$
	$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\log_e\lvert ax+b\rvert + c$

Calculus – continued

product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\times\frac{du}{dx}$
integration by parts	$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$
Euler's method	If $\frac{dy}{dx} = f(x, y), x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + h \times f(x_n, y_n)$
arc length parametric	$\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
surface area Cartesian about x-axis	$\int_{x_1}^{x_2} 2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$
surface area Cartesian about y-axis	$\int_{y_1}^{y_2} 2\pi x\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$
surface area parametric about x-axis	$\int_{t_1}^{t_2} 2\pi y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
surface area parametric about y-axis	$\int_{t_1}^{t_2} 2\pi x\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

Kinematics

acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$	
constant acceleration formulas	$v = u + at$	$s = ut + \frac{1}{2}at^2$
	$v^2 = u^2 + 2as$	$s = \frac{1}{2}(u+v)t$

Vectors in two and three dimensions

$\underset{\sim}{r}(t) = x(t)\underset{\sim}{i} + y(t)\underset{\sim}{j} + z(t)\underset{\sim}{k}$	$\lvert \underset{\sim}{r}(t) \rvert = \sqrt{x(t)^2 + y(t)^2 + z(t)^2}$	
	$\dot{\underset{\sim}{r}}(t) = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$	
for $\underset{\sim}{r}_1 = x_1\underset{\sim}{i} + y_1\underset{\sim}{j} + z_1\underset{\sim}{k}$ and $\underset{\sim}{r}_2 = x_2\underset{\sim}{i} + y_2\underset{\sim}{j} + z_2\underset{\sim}{k}$	vector scalar product $\underset{\sim}{r}_1 . \underset{\sim}{r}_2 = \lvert \underset{\sim}{r}_1 \rvert \lvert \underset{\sim}{r}_2 \rvert \cos(\theta) = x_1x_2 + y_1y_2 + z_1z_2$	
	vector cross product $\underset{\sim}{r}_1 \times \underset{\sim}{r}_2 = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = (y_1z_2 - y_2z_1)\underset{\sim}{i} + (x_2z_1 - x_1z_2)\underset{\sim}{j} + (x_1y_2 - x_2y_1)\underset{\sim}{k}$	
vector equation of a line	$\underset{\sim}{r}(t) = \underset{\sim}{r}_1 + t\underset{\sim}{r}_2 = (x_1 + x_2t)\underset{\sim}{i} + (y_1 + y_2t)\underset{\sim}{j} + (z_1 + z_2t)\underset{\sim}{k}$	
parametric equation of a line	$x(t) = x_1 + x_2t \qquad y(t) = y_1 + y_2t \qquad z(t) = z_1 + z_2t$	
vector equation of a plane	$\underset{\sim}{r}(s,t) = \underset{\sim}{r}_0 + s\underset{\sim}{r}_1 + t\underset{\sim}{r}_2$ $= (x_0 + x_1s + x_2t)\underset{\sim}{i} + (y_0 + y_1s + y_2t)\underset{\sim}{j} + (z_0 + z_1s + z_2t)\underset{\sim}{k}$	
parametric equation of a plane	$x(s,t) = x_0 + x_1s + x_2t, \ \ y(s,t) = y_0 + y_1s + y_2t, \ \ z(s,t) = z_0 + z_1s + z_2t$	
Cartesian equation of a plane	$ax + by + cz = d$	

Circular functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$	$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$
$\sin(2x) = 2\sin(x)\cos(x)$	
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$
$\sin^2(ax) = \frac{1}{2}(1 - \cos(2ax))$	$\cos^2(ax) = \frac{1}{2}(1 + \cos(2ax))$

End of formula sheet